## $\boxed{\text{n-te røtter av komplekse tall}}$

La $z \neq 0$ være et komplekst tall. La $n$ være et naturlig tall.
Med en n-te rot til $z$ menes et komplekst tall $w$
slik at

$$w^n = \underbrace{w \cdot w \cdots w}_{n \text{ stk}} = z$$

### Hvordan finne de $n$ ulike $n$-te røttene til et komplekst tall $z \neq 0$

1. Skriv $z$ på polar form $z = re^{i\theta}$ med $\theta \in [0, 2\pi)$.
   Tegn figur.

2. Finn den prinsipale $n$-te roten til $z$:

$$w_0 = \sqrt[n]{r}\, e^{i(\theta/n)}$$

3. Finn $w_+ = e^{i(2\pi/n)}$

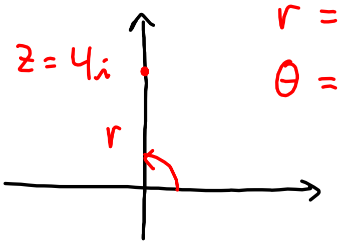
4. Finn de resterende røttene $w_1, w_2, \ldots, w_{n-1}$
   ved å gange $w_0$ med $w_+$ om igjen og om igjen,
   inntil du har $n$ ulike røtter.

eks. Skal finne annenrøttene (kvadratrøttene) til

$$z = 4i = 0 + 4i$$

① Tegner figur:

$$z = re^{i\theta} = 4e^{i(\pi/2)}$$




② Prinsipal rot: $w_0 = \sqrt[n]{r}\, e^{i(\theta/n)} = \sqrt[2]{r}\, e^{i(\theta/2)}$

$$= \sqrt[2]{4}\, e^{i(\pi/4)} = 2e^{i(\pi/4)}$$

③ $w_+ = e^{i(2\pi/n)} = e^{i(2\pi/2)} = e^{i\pi}$

④ Neste rot: $w_1 = w_+ w_0 = e^{i\pi} \cdot 2e^{i(\pi/4)}$

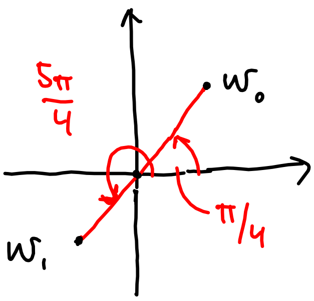
$$= 2 \cdot e^{i\pi} \cdot e^{i(\pi/4)}$$

$$\boxed{e^a e^b = e^{a+b}}$$

$$= 2 \cdot e^{i\pi + i(\pi/4)} = 2e^{i(\pi + \frac{\pi}{4})}$$

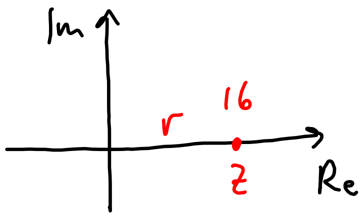
$$= 2e^{i(5\pi/4)}$$

Figur:



eks. 2 Finn fjerderøttene til $z = 16$

① Figur:



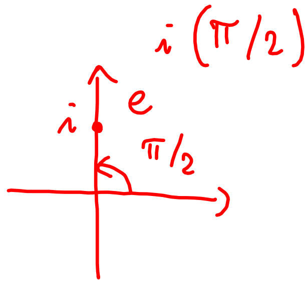

$$z = re^{i\theta} = 16e^{i\cdot 0}$$

② Prinsipal rot: $w_0 = \sqrt[4]{r}\, e^{i(\theta/4)} = \sqrt[4]{16}\, e^{i(0/4)}$

$$= 2e^{i\cdot 0} = 2e^{0} = 2$$

③ $w_+ = e^{i(2\pi/4)}$

$$= e^{i(\pi/2)} = i$$



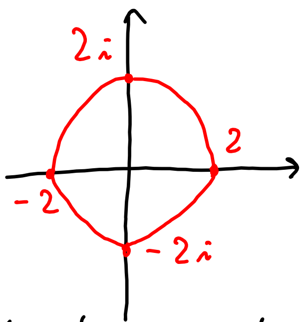

④ $w_1 = w_+ w_0 = i \cdot 2 = \underline{\underline{2i}}$

$w_2 = w_+ w_1 = i(2i) = \underline{\underline{-2}}$

$w_3 = w_+ w_2 = i(-2) = \underline{\underline{-2i}}$

Har fra før:

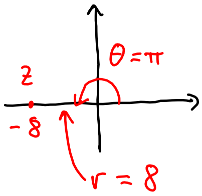
$w_0 = \underline{\underline{2}}$




røttene ligger jevnt fordelt på en sirkel om 0.

eks. 3 Skal finne tredjerøttene til $z = -8$

① Tegner figur:



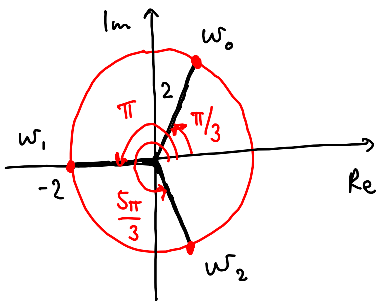

$$z = re^{i\theta} = 8e^{i\pi}$$

② Prinsipal rot:
$$w_0 = \sqrt[n]{r}\, e^{i(\theta/n)} = \sqrt[3]{8}\, e^{i(\pi/3)}$$
$$= \underline{\underline{2e^{i(\pi/3)}}} = 2\Big(\underbrace{\cos\tfrac{\pi}{3}}_{\frac{1}{2}} + i \underbrace{\sin\tfrac{\pi}{3}}_{\frac{\sqrt{3}}{2}}\Big)$$

③
$$w_+ = e^{i(2\pi/n)}$$
$$= e^{i(2\pi/3)} = e^{i\left(\frac{2}{3}\pi\right)}$$

④
$$w_1 = w_+ w_0 = e^{i\left(\frac{2}{3}\pi\right)} \cdot 2e^{i\left(\frac{1}{3}\pi\right)}$$
$$= 2 \cdot e^{i\left(\frac{2}{3}\pi\right) + i\left(\frac{1}{3}\pi\right)} = \underline{\underline{2 \cdot e^{i\pi}}} = -2$$

$$w_2 = w_+ w_1 = e^{i\left(\frac{2}{3}\pi\right)} \cdot 2e^{i\pi}$$
$$= \underline{\underline{2e^{i\left(\frac{5}{3}\pi\right)}}}$$




$$w_0 = 2e^{i\frac{\pi}{3}}$$
$$w_1 = 2e^{i\pi} = -2$$
$$w_2 = 2e^{i\left(\frac{5\pi}{3}\right)}$$

Hvis du vil skrive røttene på rektangulær form $a + bi$, så kan du regne slik:

$$w_2 = 2e^{i\left(\frac{5\pi}{3}\right)} = 2\left[\cos\left(\frac{5\pi}{3}\right) + i \cdot \sin\left(\frac{5\pi}{3}\right)\right]$$

$\boxed{\text{Se regning for } w_0}$

$$= 2\left[\frac{1}{2} + i\left(-\frac{\sqrt{3}}{2}\right)\right]$$
$$= \underline{\underline{1 - \sqrt{3}\, i}}$$