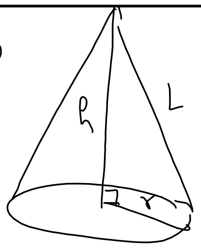
7.1.5



$L = 9$

Hva er maks volum?

Husk: $V = \dfrac{\pi r^2 h}{3}$

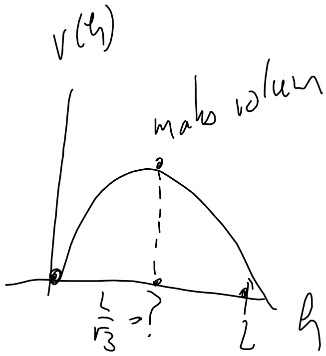
Merk 2: $h^2 + r^2 = L^2 \Rightarrow r^2 = L^2 - h^2$

Så $V(h) = \dfrac{\pi}{3}(L^2 - h^2)h = \dfrac{\pi}{3}(L^2 h - h^3)$

$V'(h) = \dfrac{\pi}{3}(L^2 - 3h^2) = 0$

$L^2 = 3h^2 \Rightarrow h^2 = \dfrac{L^2}{3}$

$\Rightarrow h = \dfrac{L}{\sqrt{3}}$.

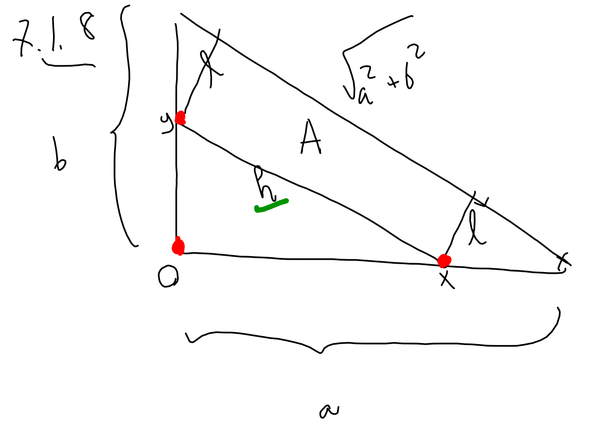


Så maks volum er $V(h) = V\left(\dfrac{L}{\sqrt{3}}\right) = \dfrac{\pi}{3}\left(L^3 \cdot \dfrac{1}{\sqrt{3}} - \dfrac{L^3}{3\sqrt{3}}\right)$

$= \dfrac{\pi}{3}\left(\dfrac{3L^3}{3\sqrt{3}} - \dfrac{L^3}{3\sqrt{3}}\right) = \dfrac{\pi}{3}\left(\dfrac{2L^3}{3\sqrt{3}}\right)$

Husk $L = 9$

$= \dfrac{\pi}{\cancel{3}}\left(\dfrac{2 \cdot 9^3}{\cancel{3}\,3\sqrt{3}}\right) = \dfrac{\pi}{\sqrt{3}}\, 2 \cdot 81$



Maksimer Arealet til A.

$$A = hl$$

Merk: Hypotenus er $\sqrt{a^2+b^2}$

Merk: Trekanten Oxy er formlik med den store trekanten (altså du kan skalere den lille til å bli den store)

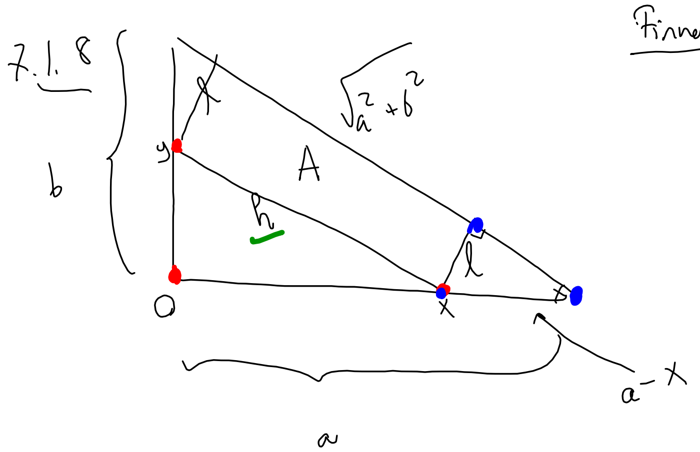
lang kateter i den lille

lang kateter i den store

hypotenus i den lille

hyp i den store

Så $\frac{x}{a} = \frac{h}{\sqrt{a^2+b^2}}$ $\Rightarrow$ $h = \frac{x\sqrt{a^2+b^2}}{a}$



Finner l: Trekanten er formlik m/ den store.

Så har at

$$\frac{a-x}{\sqrt{a^2+b^2}} = \frac{l}{b}$$

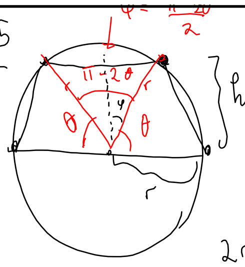
Så $l = \frac{b(a-x)}{\sqrt{a^2+b^2}}$

Så $A(x) = \frac{b}{a}x(a-x) = \frac{b}{a}(ax-x^2)$

$A'(x) = \frac{b}{a}(a-2x) = 0 \Rightarrow a = 2x \Rightarrow x = \frac{a}{2}$

Så $A_{maks} = A\left(\frac{a}{2}\right) = \frac{b}{\cancel{a}}\frac{\cancel{a}}{2}\cdot\frac{a}{2} = \underline{\underline{\frac{ab}{4}}}.$ ✓

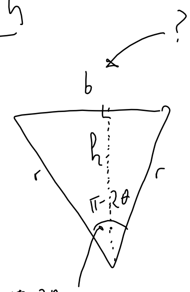
7.1.15



Trapes i sirkel. m/ hjørner på sirkelen.

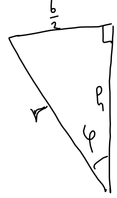
Finn maks areal.

Merk 1 Areal $= \dfrac{(\overset{2r}{\text{grunnlinje}} + \overset{b}{\text{den andre linja}})\,h}{2}$



Vil finne lengden på topplinja.

Del rektanglen i to og får



$\varphi = \dfrac{\pi-2\theta}{2}$

Dermed: $\cos\varphi = \dfrac{h}{r}$ $\quad h = r\cos\varphi \quad$ og $\quad \sin\varphi = \dfrac{b/2}{r} \Rightarrow b = 2r\sin\varphi$

Dermed $$A(\varphi) = \frac{(2r + 2r\sin\varphi)\, r\cos\varphi}{2} = r^2\underline{(1+\sin\varphi)\cos\varphi}$$

A er en funksjon av φ og vi maksimerer ved å derivere.

$$\frac{2(r+r\sin\varphi)\, r\cos\varphi}{2}$$

$$\frac{dA(\varphi)}{d\varphi} = r^2\left(\cos\varphi\cdot\cos\varphi - \sin\varphi(1+\sin\varphi)\right)$$
$$= r^2\left(\cos^2\varphi - \sin\varphi - \sin^2\varphi\right)$$
$$= r^2\left(1-\sin^2\varphi - \sin\varphi - \sin^2\varphi\right)$$
$$= r^2\left(-2\sin^2\varphi - \sin\varphi + 1\right) = 0$$

Bruk at
$\cos^2\varphi + \sin^2\varphi = 1$
$\cos^2\varphi = 1 - \sin^2\varphi$

Så må løse $-2(\sin\varphi)^2 - (\sin\varphi) + 1 = 0$

Så $\sin\varphi = \dfrac{1\pm\sqrt{1+8}}{-4} = \dfrac{1\pm 3}{-4} = \dfrac{1}{2}$ eller -1

svarer til $\varphi = \dfrac{\pi}{6}$ eller $\varphi = \dfrac{3\pi}{2}$

eneste lovlige løsning.

Dermed er maks areal $A = r^2\left(1+\frac{1}{2}\right)\frac{\sqrt{3}}{2}$
$= r^2\left(\frac{3}{2}\right)\frac{\sqrt{3}}{2} = \underline{\underline{\dfrac{r^2\, 3\sqrt{3}}{4}}}$

7.6.7 a) $\frac{1+x}{1+x^2} = 2\arctan x$

V.a. har én reell løsning $x = x_0$

og at $\frac{1}{3}\sqrt{3} < x_0 < 1$.

Lager hjelpefunksjon. $g(x) = \frac{1+x}{1+x^2} - 2\arctan x$

Bruker skjæringssetningen: har $g\left(\frac{\sqrt{3}}{3}\right) = \frac{1+\sqrt{3}/3}{1+\frac{1}{3}} - 2\arctan\left(\frac{\sqrt{3}}{3}\right)$

$= \frac{1+\sqrt{3}/3}{4/3} - 2\cdot\frac{\pi}{6} > 0$

Husk $\arctan x = y \Leftrightarrow \tan y = x$ | $\arctan\frac{\sqrt{3}}{3} = \frac{\pi}{6}$

$\tan\frac{\pi}{3} = \frac{\sqrt{3}/2}{1/2} = \sqrt{3}$

$\tan\frac{\pi}{6} = \frac{1/2}{\sqrt{3}/2} = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$

og $g(1) = \frac{1+1}{1+1} - 2\arctan 1$

$= 1 - 2\frac{\pi}{4} = 1 - \frac{\pi}{2} < 0$

Så ved skjæringssetningen finnes $x_0 \in \left(\frac{\sqrt{3}}{3}, 1\right)$ slik at $g(x_0) = 0$.

b) $\varphi(x) = \frac{\arctan x}{(1+x)^2}$ Finn avtagende/stigende.

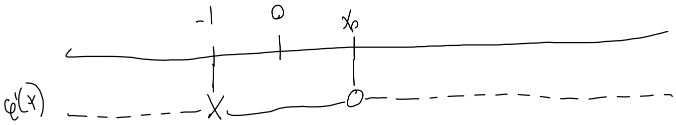
Husk $[\arctan x]' = \frac{1}{1+x^2}$

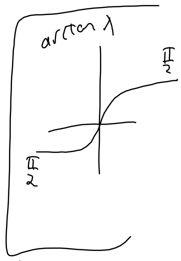
Deriverer og finner når $\varphi'(x) > 0$ og < 0.

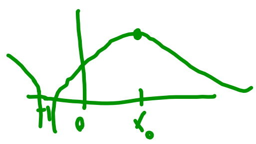
$$\varphi'(x) = \frac{\frac{1}{1+x^2}\cdot(1+x)^2 - 2\arctan x\cdot(1+x)}{(1+x)^4} = \frac{\frac{1}{1+x^2}(1+x) - 2\arctan x}{(1+x)^3}$$

$$= \frac{1}{(1+x)^3}\underbrace{\left(\frac{1+x}{1+x^2} - 2\arctan x\right)}_{\text{uttrykket fra a)}} = 0$$

har nøyaktig ett nullpunkt $\left(\frac{\sqrt{3}}{3}, 1\right)$







Konklusjon

Fra fortegnslinjen ser vi at globalt toppunkt i x_0.