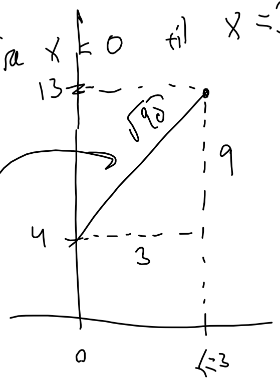
8.6.11 a)

$y = 3x+4$ fra $x=0$ til $x=3$

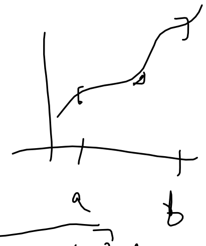
$[3x+4]' = 3$

vil finne lengden på denne



$$L = \int_0^3 \sqrt{1+f'(t)^2}\,dt = \int_0^3 \sqrt{1+3^2}\,dt = \int_0^3 \sqrt{10}\,dt = \sqrt{10}\cdot 3$$

Buelengde



$$L = \int_a^b \sqrt{1+f'(t)^2}\,dt$$

b) $f(x) = \frac{x^2}{2} - \frac{1}{4}\ln x$ fra $x=1$ til $x=e$ (tegn denne)

$f'(x) = x - \frac{1}{4}\frac{1}{x}$

$$f'(x)^2 = x^2 - 2\cdot x\cdot\frac{1}{4}\frac{1}{x} + \frac{1}{16}\frac{1}{x^2} = x^2 - \frac{1}{2} + \frac{1}{16}\frac{1}{x^2}$$

Da er

$$L = \int_1^e \sqrt{\underbrace{x^2 + \frac{1}{2} + \frac{1}{16}\frac{1}{x^2}}_{\left(x+\frac{1}{4}\frac{1}{x}\right)^2}}\,dx = \int_1^e x + \frac{1}{4}\frac{1}{x} = \left[\frac{1}{2}x^2 + \frac{1}{4}\ln x\right]_1^e$$

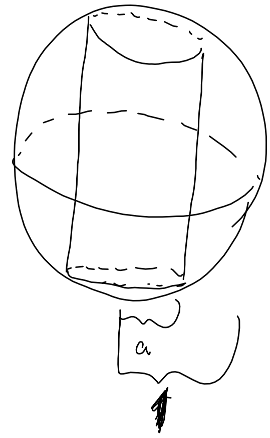
$$= \frac{1}{2}e^2 + \underbrace{\frac{1}{4} - \frac{1}{2}} = \frac{1}{2}e^2 - \frac{1}{4}$$

8.6.15

Kule m/ radius 1

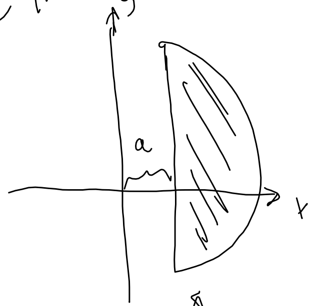
borer sylinderformet hull.

Sylinderen har radius $a < 1$.



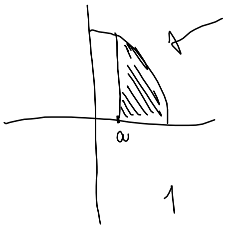
Finn Volum.

Ser denne fra siden:



[Paisrand Roter denne rundt y-aksen, og vi får figuren over.]

Ikke en funksjonsgraf!



Roter heller denne rundt y-aksen

$y = \sqrt{1-x^2}$

Omdr. legemet vi får da er halve svaret.

$$\frac{V}{2} = \int_a^1 2\pi x \cdot f(x)\, dx = \pi \int_a^1 2x\sqrt{1-x^2}\, dx$$

$$= -\pi \int_a^1 \underbrace{-2x}_{u'} \underbrace{\sqrt{1-x^2}}_{u}\, dx = \pi \int_1^a \underbrace{-2x}_{u'} \underbrace{\sqrt{1-x^2}}_{u}\, dx$$

$u = 1-x^2$

$du = -2x\, dx$

$u(1) = 1-1 = 0$

$u(a) = 1-a^2$

$$= \pi \int_0^{1-a^2} \sqrt{u}\, du = \pi \left[\frac{2}{3} u^{\frac{3}{2}} \right]_0^{1-a^2}$$

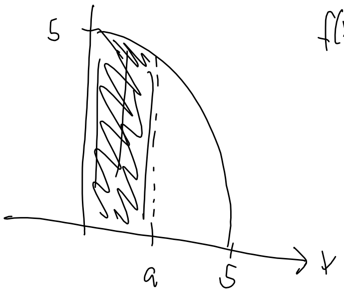
$$= \frac{2\pi}{3} \left(1-a^2\right)^{3/2}$$

Så $V = \underline{\underline{\frac{4\pi}{3}(1-a^2)^{3/2}}}$

8.6.26 $a \in [0,5]$

Finn volumet av omdr. legemet rundt x-aksen.

$f(x) = \sqrt{25 - x^2}$



a)

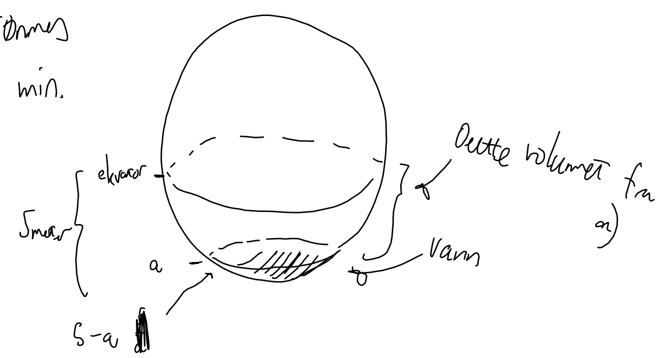
$$V(a) = \pi \int_0^a f(x)^2 \, dx$$

$$= \pi \int_0^a 25 - x^2 \, dx = \pi \left[25x - \frac{1}{3}x^3 \right]_0^a$$

$$= \pi \left(25a - \frac{1}{3}a^3 \right)$$

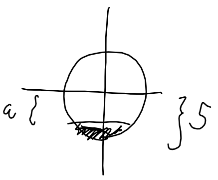
b) Når vanndybden er 2-meter tømmes tanken m/ $\frac{1}{2}$ kubikkm pr min.

Hvor fort avtar vanndybden?



Volum luft v/ h meter vann = $V(5-h)$

Høyden h avhenger av tiden t, så skriver $h(t)$.



Så volum v/ tiden t er $V(5-h(t))$.

Vi skal finne $h'(t)$ når det er 2 m/ vann.

Har at $\quad V(t)' = V(5-h(t))' = -h'(t) \cdot V'(5-h(t))$

$$h'(t) = \frac{-V(t)'}{V'(\underbrace{5-h(t)}_{3})} = \frac{-\frac{1}{2}\frac{m^3}{min}}{16\pi} = \frac{-1}{32\pi}$$

$$V'(a) = \pi(25 - a^2)$$

$$V'(3) = \pi(25 - 9) = 16\pi$$

Kjerneregel $\quad \frac{\partial h}{\partial t} = \frac{\frac{\partial V}{\partial t}}{\frac{\partial V}{\partial a}}$

høyde-endring

volum-endr pr tid

volum-endr pr meter

9.1.11

$$I = \int \frac{\overbrace{x^2 \arctan x}^{v}}{\underbrace{1+x^2}_{u'}}\,dx$$

$$\boxed{\int u'v = uv - \int uv'}$$

$$u' = \frac{x^2}{1+x^2} \qquad u = \int \frac{x^2}{1+x^2}\,dx = \int \frac{1+x^2-1}{1+x^2}\,dx$$

$$\left(I = uv - \int uv'\right) \qquad = \int 1 - \frac{1}{1+x^2}\,dx = x - \arctan x$$

$$I = (x - \arctan x)\cdot \arctan x - \int (x-\arctan x)\cdot \frac{1}{1+x^2}\,dx$$

$$= (x-\arctan x)\arctan x - \underbrace{\int \frac{x}{1+x^2}\,dx}_{I_1} + \underbrace{\int \frac{\arctan x}{1+x^2}\,dx}_{I_2}$$

$$\int_a^b u'(t)\,v(t)\,dt = \Big[u(t)v(t)\Big]_a^b - \int_a^b u(t)\,v'(t)\,dt$$

Merk $\left[f(t)^2\right]' = 2f(t)f'(t)$

Har $I_1 = \frac{1}{2}\int \frac{2x}{1+x^2}\,dx = \frac{1}{2}\ln(1+x^2)$

$$I_2 = \int \underbrace{\frac{1}{1+x^2}}_{f'}\cdot \underbrace{\arctan x}_{f}\,dx = \frac{1}{2}\int \underbrace{2}_{2}\,\underbrace{\frac{1}{1+x^2}}_{f'}\,\underbrace{\arctan x}_{f}\,dx = \frac{1}{2}(\arctan x)^2 + C$$

Så $I = x\arctan x - (\arctan x)^2 - \frac{1}{2}\ln(1+x^2) + \frac{1}{2}(\arctan x)^2 + C$

$$= x\arctan x - \frac{1}{2}(\arctan x)^2 - \frac{1}{2}\ln(1+x^2) + C$$

9.2.1

c) $I = \int \frac{x}{\sqrt{x}+1}\, dx$

$u = \sqrt{x}$

eller $\underbrace{u = \sqrt{x} + 1}$

$\Leftrightarrow \sqrt{x} = u - 1$

$\Leftrightarrow x = (u-1)^2 = u^2 - 2u + 1$

$dx = 2u\,du - 2\,du$

$\boxed{dx = (2u-2)\,du}$

$$I = \int \frac{u^2-2u+1}{u}(2u-2)\,du = 2\int \frac{(u^2-2u+1)(u-1)}{u}\,du$$

$$= 2\int \frac{(u-1)^3}{u}\,du = 2\int \frac{u^3 - 3u^2 + 3u - 1}{u}\,du$$

$$= 2\int u^2 - 3u + 3 - \frac{1}{u}\,du = 2\left[\frac{1}{3}u^3 - \frac{3}{2}u^2 + 3u - \ln u + C\right]$$

$$= \frac{2}{3}u^3 - 3u^2 + 6u - 2\ln u + C'$$

$$= \frac{2}{3}(\sqrt{x}+1)^3 - 3(\sqrt{x}+1)^2 + 6(\sqrt{x}+1) - 2\ln(\sqrt{x}+1) + C'$$

$$\int_0^3 \arctan \underbrace{\sqrt{x}}_{u}\, dx$$

$u = \sqrt{x}$

$u^2 = x$

$2u\, du = dx$

$u = \sqrt{x}$

$\Downarrow$

nye grenser:

$u(0) = 0 \quad u(3) = \sqrt{3}$

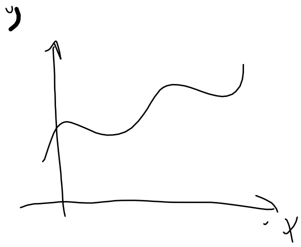
$$= 2\int_0^{\sqrt{3}} \arctan u \cdot u\, du$$

$$= 2\left[\left[\arctan u \cdot \frac{1}{2}u^2\right]_0^{\sqrt{3}} - \int_0^{\sqrt{3}} \frac{1}{1+u^2} \cdot \frac{1}{2}u^2\, du \right]$$

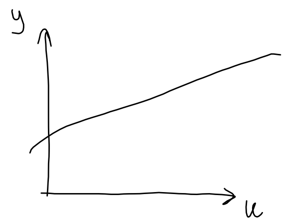
$$= 2\left[\frac{3}{2} \cdot \frac{\pi}{3} - \frac{1}{2}\int_0^{\sqrt{3}} \frac{u^2}{1+u^2}\, du \right] = \pi - \int_0^{\sqrt{3}} 1 - \frac{1}{1+u^2}\, du$$

$$= \pi - \Big[u - \arctan u\Big]_0^{\sqrt{3}} = \pi - \left(\sqrt{3} - \frac{\pi}{3}\right)$$

$$= \frac{4\pi}{3} - \sqrt{3} \quad //$$



$u = f(x)$



$x + iy$

$re^{i\theta}$

$$z = \underbrace{x^2 + y^2}_{r^2} = r^2$$