9.3.5 f $\displaystyle\int \frac{3x^2+x}{(x-1)(x+1)^2}\,dx$

$$\frac{3x^2+x}{(x-1)(x+1)^2} = \frac{A}{x-1} + \frac{B}{x+1} + \frac{C}{(x+1)^2}$$

$$3x^2+x = A(x+1)^2 + B(x+1)(x-1) + C(x-1)$$

Setter $x=1$

$4 = 4A + 0 + 0$

Så $A = 1$

$x = -1$

$2 = C(-2) \Rightarrow$ så $C = -1$.

Sett $x = 0$

$0 = A - B - C$ $\quad B = 2$

$= 1 - B + 1 = 2 - B$

Så $\displaystyle\frac{3x^2+x}{(x-1)(x+1)^2} = \frac{1}{x-1} + \frac{2}{x+1} - \frac{1}{(x+1)^2}$

Alle ledd enkle å integrere.

Så $\displaystyle\int \frac{3x^2+x}{(x-1)(x+1)^2}\,dx = \ln|x-1| + 2\ln|x+1| + \frac{1}{x+1} + C$

9.3.21 a) $I = \int \frac{u+2}{u^2+2u+5}\, du$

$\frac{-b \pm \sqrt{b^2-4ac}}{2a}$

Er irredusibel om $b^2 - 4ac < 0$:

$4 - 4 \cdot 1 \cdot 5 < 0$

Så ingen reelle røtter.

Derivert til nevner er $2u+2$.

Merk at $\frac{u+2}{u^2+2u+5} = \frac{1}{2} \frac{2u+4}{u^2+2u+5} = \frac{1}{2} \frac{\overbrace{2u+2}^{f'}+2}{u^2+2u+5}$

$= \frac{1}{2} \frac{2u+2}{u^2+2u+5} + \frac{1}{u^2+2u+5}$

$(ab)^2 = a^2 \cdot b^2$

Husk

$(\arctan x)' = \frac{1}{1+x^2}$

Så $I = \frac{1}{2} \int \underbrace{\frac{2u+2}{u^2+2u+5}}\, du + \int \frac{1}{u^2+2u+5}\, du$

$v = u^2+2u+5$

$dv = 2u+2\, du$

$= \frac{1}{2} \int \frac{1}{v}\, dv + \int \frac{1}{u^2+2u+1+4}\, du$

$\frac{1}{4}(u+1)^2 = \left(\frac{1}{2}\right)^2 (u+1)^2$

$= \left(\frac{u+1}{2}\right)^2$

$= \frac{1}{2} \ln|v| + \int \frac{1}{4+(u+1)^2}\, du = \frac{1}{2} \ln|v| + \int \frac{1/4}{1+\left(\frac{u+1}{2}\right)^2}\, du$

$= \frac{1}{2} \ln|v| + \frac{1}{4} \int \frac{1}{1+\left(\frac{u+1}{2}\right)^2}\, du$

Sett $v = \frac{u+1}{2}$ $\quad dv = \frac{1}{2}\, du$

$du = 2\, dv$

Så $= \frac{1}{2} \ln|v| + \frac{1}{4} \int \frac{2\, dv}{1+v^2}$

$= \frac{1}{2} \ln|v| + \frac{1}{2} \arctan v \quad = \frac{1}{2} \ln|u^2+2u+5| + \frac{1}{2} \arctan\left(\frac{u+1}{2}\right) + C$

B) Finn A, B, C s.a.

$$\frac{1}{u(u^2+2u+5)} = \frac{A}{u} + \frac{Bu+C}{u^2+2u+5}.$$

$$1 = A(u^2+2u+5) + (Bu+C)u$$

$$= u^2(A+B) + u(2A+C) + 5A$$

$A + B = 0$ $\quad$ $B = -\frac{1}{5}$

$2A + C = 0$ $\quad$ $C = -\frac{2}{5}$

$5A = 1 \Rightarrow A = \frac{1}{5}$

$$= \frac{1}{5}\frac{1}{u} - \frac{1}{5}\frac{u+2}{u^2+2u+5}$$

nov 25-08:41

c) $\displaystyle\int \frac{\tan x}{\cos^2 x + 2\cos x + 5}\,dx \qquad \tan x = \frac{\sin x}{\cos x}$

$$= -\int \frac{-\sin x}{\cos x(\cos^2 x + 2\cos x + 5)}\,dx$$

Setter $u = \cos x$
så er $du = -\sin x\,dx$

$$= -\int \frac{1}{u(u^2+2u+5)}\,du$$

(*)

$$= -\int \frac{1}{5}\frac{1}{u} - \frac{1}{5}\underbrace{\frac{u+2}{u^2+2u+5}}_{\text{gjør denne først, kall den } I'}\,du$$

$$\frac{\overset{f'}{2u+2}}{\underset{= f}{u^2+2u+5}} + \frac{2}{u^2+2u+5}$$

$$I' = \int \frac{u+2}{u^2+2u+5}\,du = \frac{1}{2}\int \frac{2u+2+2}{u^2+2u+5}\,du$$

$$= \frac{1}{2}\ln(u^2+2u+5) + \int \frac{1}{u^2+2u+5}\,du$$

$$= -\,|| \text{———} + \int \frac{1}{(u^2+2u+1)+4}\,du$$

$$= -\,|| \text{———} \frac{1}{4}\int \frac{1}{1+\left(\frac{u+1}{2}\right)^2}\,du$$

Sett $v = \frac{u+1}{2}$ så $dv = \frac{1}{2}du$, $du = 2\,dv$

$$= -\,|| \text{———} + \frac{1}{2}\arctan\left(\frac{u+1}{2}\right) + C$$

(*) $= -\frac{1}{5}\ln|u| + \frac{1}{10}\ln(u^2+2u+5) + \frac{1}{10}\arctan\left(\frac{u+1}{2}\right) + C$

$u = \cos x$ (skriv inn)

9.3.25 a) V.a. $2+i$ er en rot i $\underbrace{z^3 - 11z + 30}_{P} = 0$.

Sett inn $2+i$ og håp på null....

Metode 1

Bedre: Husk 1 Om z er en rot av P hvor P har reelle koeffisienter, så er også $\bar{z}$ en rot.

Husk 2 Om α er en av P så er $z-\alpha$ en faktor i P.

$(z-(2+i))(z-(2-i))$ er en faktor i P.

Så vi kan skrive om

$$(z-(2+i))(z-(2-i)) = z^2 - z(2-i+2+i) + 5$$

$$f = z^2 - 4z + 5$$

Husk

$\alpha\bar{\alpha} = |\alpha|^2$

Så skriver om $f | P$:

$$z^3 - 11z + 20 : z^2 - 4z + 5 = z + 4$$
$$-(z^3 - 4z^2 + 5z)$$
$$4z^2 - 16z + 20$$
$$4z^2 - 16z + 20$$
$$0$$

Konkl. m/ at

$$P = (z^2 - 4z + 5)(z + 4)$$

B) $\int \frac{10x+3}{x^3 - 11x + 20}\,dx$ — Bruk

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$$\int 1 \cdot \ln(x^2 + 2x + 10)\, dx$$

$$= x\ln(x^2+2x+10) - \int x \cdot \frac{2x+2}{x^2+2x+10}\, dx$$

Bruk at:

$$\frac{2x^2+2x}{x^2+2x+10} = \frac{1}{2}\cdot\frac{x^2+x}{x^2+2x+10}$$

$$= \frac{1}{2}\,\frac{x^2+2x+10-x-10}{x^2+2x+10}$$

$$= \frac{1}{2}\left(1 - \frac{x+10}{x^2+2x+10}\right)$$

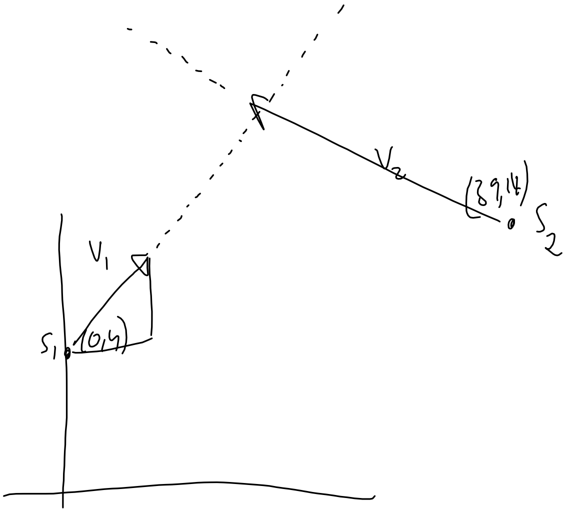
Løs dem

$$\int \frac{x+10}{x^2+2x+10} = \frac{1}{2}\,\frac{2x+20}{x^2+2x+10} = \frac{1}{2}\,\frac{2x+2+18}{x^2+2x+10}$$

$$\frac{1}{2}\,\frac{\overbrace{2x+2}^{f'}}{\underbrace{x^2+2x+10}_{f}} + \underbrace{\frac{9}{x^2+2x+10}}_{\arctan(\sim)}$$

Over til LinAlg

1.2.25 Skip 1 $(0,4)$
Skip 2 $(39,14)$



Skip 1 beveger seg i retning $(3,4)$
Skip 2 —"— $(-12,5)$

Skip 1 beveger seg i 15 knop
Skip 2 —"— 13 knop

$S_1(t) \in \mathbb{R}^2$ $\quad S_1(t) = \vec{v}_0 + kt\vec{v}_1$

Etter én tidsenhet har vi beveget oss
$s_1(t+1) - s_1(t) = k(t+1)\vec{v}_1 - kt\vec{v}_1 = k\vec{v}_1$

Så vil vi at $|k\vec{v}_1| = 15$ siden dette er farta.

$|\vec{v}_1| = 5$, så vi må sette $k = 3$.

Husk
$|\vec{v}_1| = \sqrt{a^2+b^2}$
$(v_1) = (a,b)$

Så $s_1(t) = \begin{pmatrix} 0 \\ 4 \end{pmatrix} + t \cdot 3 \begin{pmatrix} 3 \\ 4 \end{pmatrix} = \begin{pmatrix} 0 \\ 4 \end{pmatrix} + t\begin{pmatrix} 9 \\ 12 \end{pmatrix} = \begin{pmatrix} 9t \\ 4+12t \end{pmatrix}$

Gjenta for skip 2:
Der er $v_2 = (-12,5)$ og $|v_2| = \sqrt{12^2+5^2} = \sqrt{169} = 13$

Og dermed $s_2(t) = \begin{pmatrix} 39 \\ 14 \end{pmatrix} + t\begin{pmatrix} -12 \\ 5 \end{pmatrix} = \begin{pmatrix} 39 - 12t \\ 14 + 5t \end{pmatrix}$

Finner nå de har samme x-koord:
Da er $9t = 39 - 12t \Rightarrow 21t = 39 \Rightarrow t = \frac{13}{7}$

Må sjekke om dette gir samme y-koord:
$4 + 12t = 4 + 12\frac{13}{7}$ (Forskjellige, så skipene krasjer ikke)
$14 + 5t = 14 + \frac{5}{7} \cdot 13$

1.B.4 (dropper pil over vektor)

Husk $\bar a \in \mathbb{C}^n$

$$|\vec a| = \sqrt{|z_1|^2 + |z_2|^2 + \dots + |z_n|^2}$$

V.a. $|\vec x - \vec y|^2 = |\vec x|^2 - 2\,\mathrm{Re}(\vec x \cdot \vec y) + |y|^2$

$$|\vec x - \vec y|^2 = |z_1 - y_1|^2 + |z_2 - y_2|^2 + \dots + |z_n - y_n|^2$$

Husk $|z|^2 = z \cdot \bar z$

$$= (z_1 - y_1)(\bar z_1 - \bar y_1) + \dots$$

$x = (z_1, \dots, z_n)$

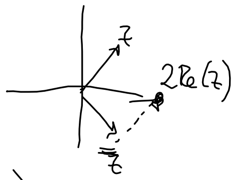
$y = (y_1, \dots, y_n)$

$$= z_1 \bar z_1 - z_1 \bar y_1 - y_1 \bar z_1 + y_1 \bar y_1 + \dots$$

$|z_1|^2$ $\quad$ $|y_1|^2$

Disse bidrar til $|\vec x|^2$ og $|\vec y|^2$

Og $-z_1 \bar y_1 - y_1 \bar z_1$ $\quad$ Merk: $\overline{z_1 \bar y_1} = \bar z_1 y_1$



$$a + \bar a = (a_1 + i a_2) + (a_1 - i a_2) = 2a_1 = 2\mathrm{Re}(a)$$

Så: $-z_1 \bar y_1 - y_1 \bar z_1 = -\mathrm{Re}(z_1 \bar y_1)$

Så når vi summerer dem får vi $-\mathrm{Re}(z_1 \bar y_1) - \mathrm{Re}(z_2 \bar y_2) = \dots$

$$= -\mathrm{Re}(z_1 \bar y_1 + z_2 \bar y_2 + \dots)$$

$$= -\mathrm{Re}(x \cdot y)$$

Husk

$v \cdot w = v_1 \bar w_1 + v_2 \bar w_2 + \dots$