MAT 1100 20.08.2024

KOMPLEKSE TALL

$$z = a + ib \qquad w = c + id$$

$$\boxed{i^2 = -1}$$

$$z + w = (a + c) + i(b + d)$$

$$z - w = (a - c) + i(b - d)$$

$$z \cdot w = (a + ib)(c + id)$$

$$= \dots$$

$$= (ac - bd) + i(ad + bc)$$

KONJUGASJON

$$z = a + ib \qquad \overline{z} = a - ib$$

EKS: $w = 5 - 4i \qquad \overline{w} = 5 + 4i$

OBS:

- $\overline{\overline{z}} = \overline{\overline{(a + ib)}} = \overline{(a - ib)} = a + ib = z$
- $\overline{z} + \overline{w} = \overline{z + w}$

  $(a - ib) + (c - id) = (a + c) - i(b + d)$
- $\overline{z} - \overline{w} = \overline{z - w}$
- $z\overline{w} = z\overline{w}$
- $\dfrac{\overline{z}}{\overline{w}} = \overline{\left(\dfrac{z}{w}\right)}$
- $w \cdot \overline{w} = (c + id)(c - id)$

  $= c^2 - icd + icd - i^2 d^2$

  $= c^2 + d^2 \in \mathbb{R}$

DIVISJON

$w \neq 0 \qquad \frac{z}{w} = \frac{a+ib}{c+id} \overset{\cdot 1}{=} \frac{(a+ib)(c-id)}{(c+id)(c-id)} \qquad 1 = \frac{\overline{w}}{w}$

$$= \frac{ac + ibc - iad - i^2 bd}{c^2+d^2}$$

$$= \frac{(ac+bd) + i(bc-ad)}{c^2+d^2}$$

$$= \frac{(ac+bd)}{c^2+d^2} + i\frac{(bc-ad)}{c^2+d^2}$$

$\mathrm{Re}\left(\frac{z}{w}\right)$ $\qquad$ $\mathrm{Im}\left(\frac{z}{w}\right)$

EKS. 3.1.3 g)

$$\frac{-5+2i}{5-4i} = \frac{(-5+2i)(5+4i)}{(5-4i)(5+4i)}$$

$$= \frac{(-5)\cdot 5 + (-5)\cdot 4i + 2i\cdot 5 + 2i\cdot 4i}{5^2+4^2}$$

$$= \frac{-25 - 20i + 10i + i^2 \cdot 8}{41}$$

$$= \frac{-25-8-10i}{41}$$

$$= -\frac{33}{41} - i\frac{10}{41}$$

OBS: • $z^{-1} = \frac{1}{z} = \frac{\bar{z}}{z\bar{z}} = \frac{a - ib}{a^2 + b^2}$

LIKNING

$$-iz + 1 = (7 + i)z$$

$$1 = (7 + i)z + iz$$

$$1 = (7 + 2i)z$$

$$z = \frac{1}{(7 + 2i)} \cdot \frac{(7 - 2i)}{(7 - 2i)}$$

$$= \frac{7 - 2i}{53}$$

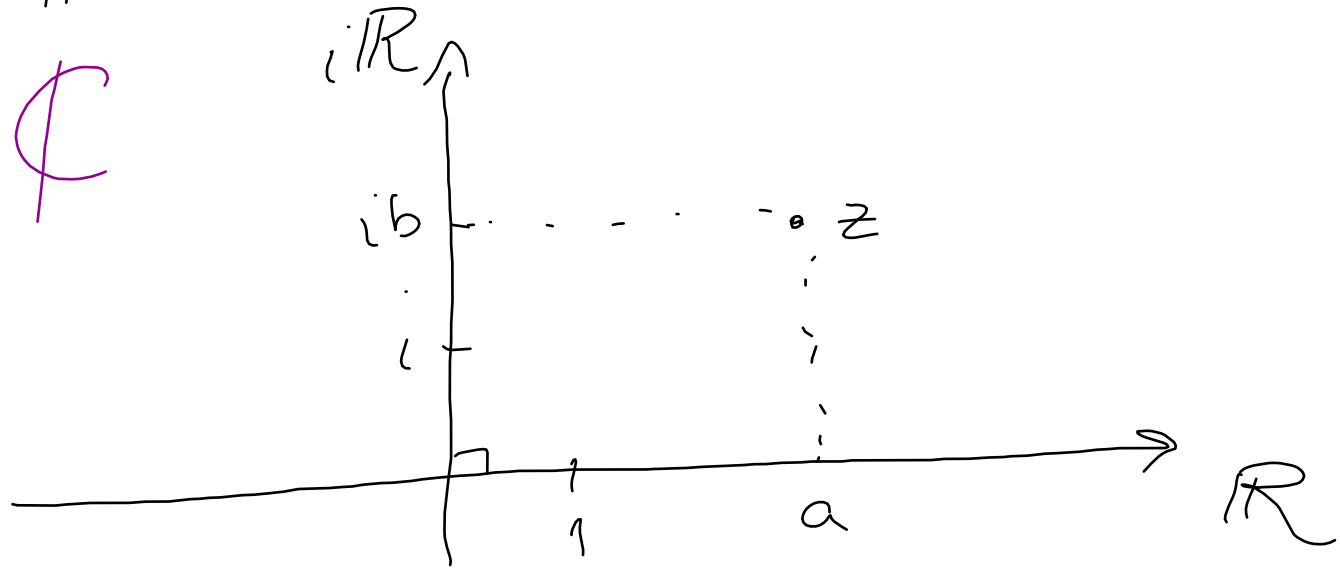
3.2. GEOMETRISK TOLKNING AV KOMPLEKSE TALL

OBS: • $z = a + ib \in \mathbb{C}$

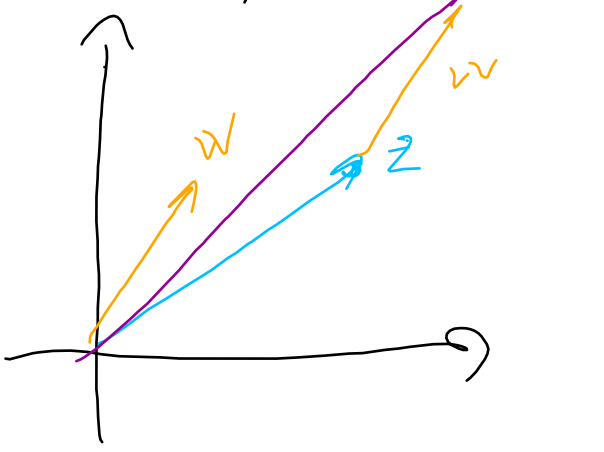
Vektor/punkt $(a, b) \in \mathbb{R}^2$

- $\mathbb{R}$ $\quad a = a + i \cdot 0 = (a, 0)$
- $i\mathbb{R}$ $\quad ib = 0 + i \cdot b = (0, b)$

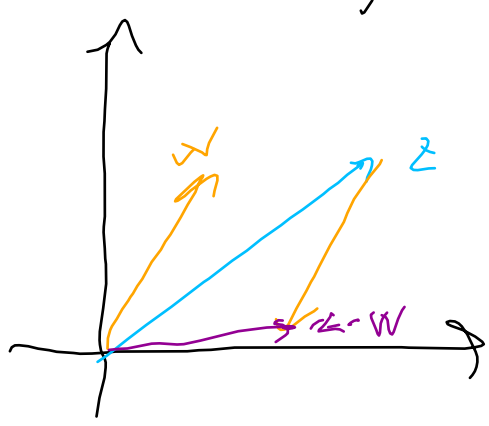
$\mathbb{R}^2 \cong \mathbb{C}$



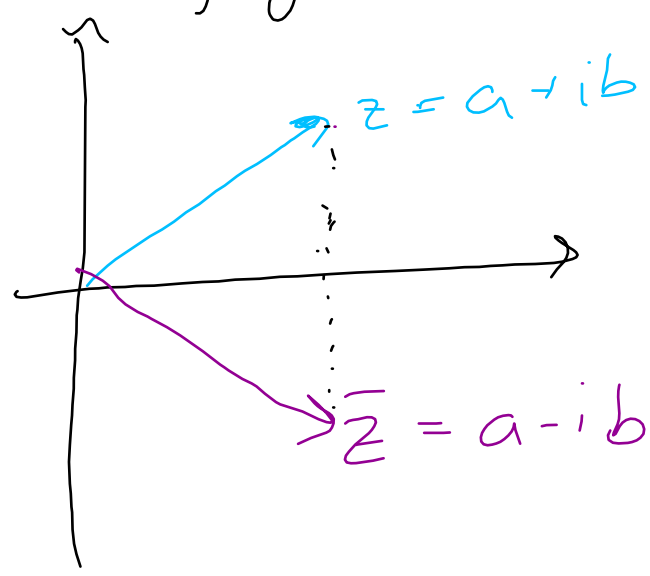
Addisjon

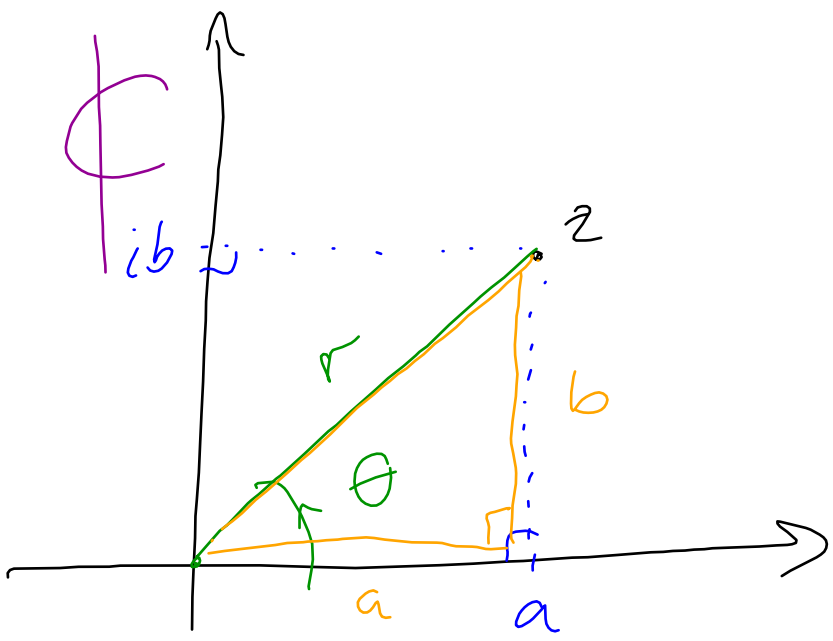


Subtraksjon



Konjugasjon





KARTESISK FORM

(a, b)

$z = a + ib$

$a = \mathrm{Re}(z)$ $\quad a, b \in \mathbb{R}$

$b = \mathrm{Im}(z)$

OVERGANGSFORMLER

Pytagoras: $r = \sqrt{a^2 + b^2}$

θ: $\cos\theta = \frac{a}{r}$ $\quad a = r\cos\theta$

$\sin\theta = \frac{b}{r}$ $\quad b = r\sin\theta$

POLAR FORM

(r, θ)

$z = r(\cos\theta + i\sin\theta)$

Modulus $r \in [0, \infty)$

$\|z\|$ absoluttverdi

Argument $\theta \in [0, 2\pi)$

$\theta = \theta + 2\pi \cdot k, \; k \in \mathbb{Z}$

EKS: Polarform til kartesisk form

$r = 2 \qquad \theta = \frac{5\pi}{3}$

TRIGONOMETRI

Skal finne $z = a + ib$

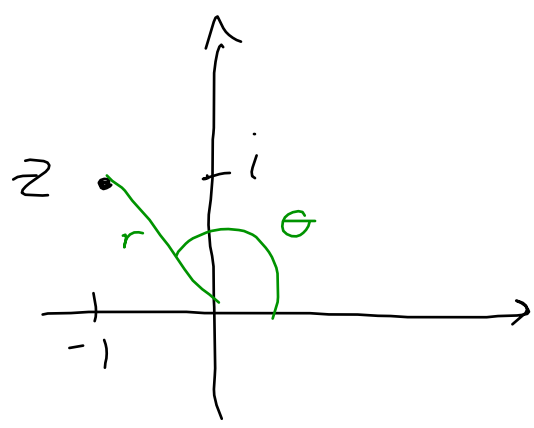
$a = r\cos\theta = 2\cos\frac{5\pi}{3} = 2 \cdot \frac{1}{2} = 1$

$b = r\sin\theta = 2\sin\frac{5\pi}{3} = 2\left(-\frac{\sqrt{3}}{2}\right) = -\sqrt{3}$

$$\underline{z = 1 - \sqrt{3}i}$$

EKS Kartesisk form til polaform

$z = -1 + i \qquad a = -1 \qquad b = 1$



$r = \sqrt{a^2 + b^2} = \sqrt{(-1)^2 + 1^2} = \sqrt{2}$

θ: $\cos\theta = \frac{-1}{\sqrt{2}} = \frac{-\sqrt{2}}{2}$

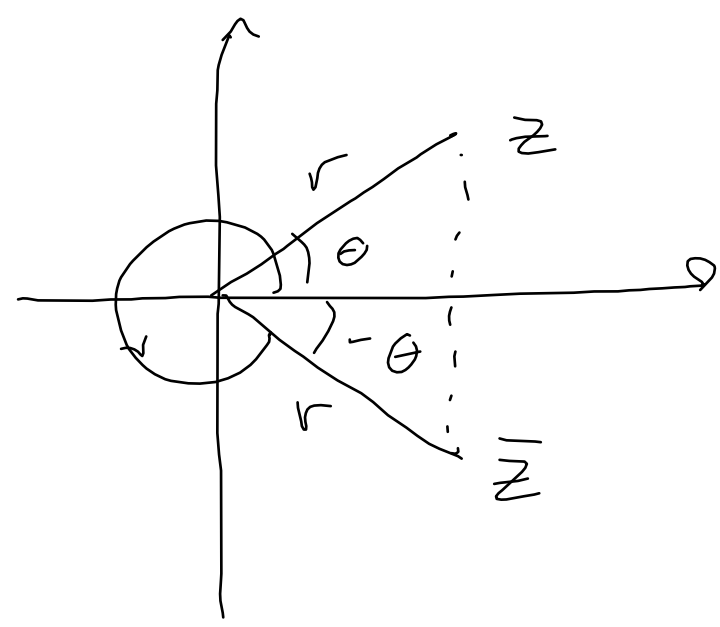
$\sin\theta = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

$\theta = \frac{3\pi}{4}$

$$\underline{z = \sqrt{2}\left(\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}\right)}$$

OBS:

- $z \cdot \bar{z} = a^2 + b^2 = r^2 = |z|^2$
- $\bar{z} = r(\cos(-\theta) + i\sin(-\theta))$ | $z = r(\cos\theta + i\sin\theta)$



TEOREM 3.2.3

La $z_1 = a_1 + ib_1 = r_1(\cos\theta_1 + i\sin\theta_1)$

og $z_2 = a_2 + ib_2 = r_2(\cos\theta_2 + i\sin\theta_2)$

vore to komplekse tall med produktet

$$z = z_1 \cdot z_2 = (a_1a_2 - b_1b_2) + i(a_1b_2 + a_2b_1)$$

Da har z modulus $r_1 \cdot r_2$

og argument $\theta_1 + \theta_2$

EKS: $z_1 = 1 - i\sqrt{3}$ $\quad (r_1, \theta_1) = (2, \frac{5\pi}{3})$

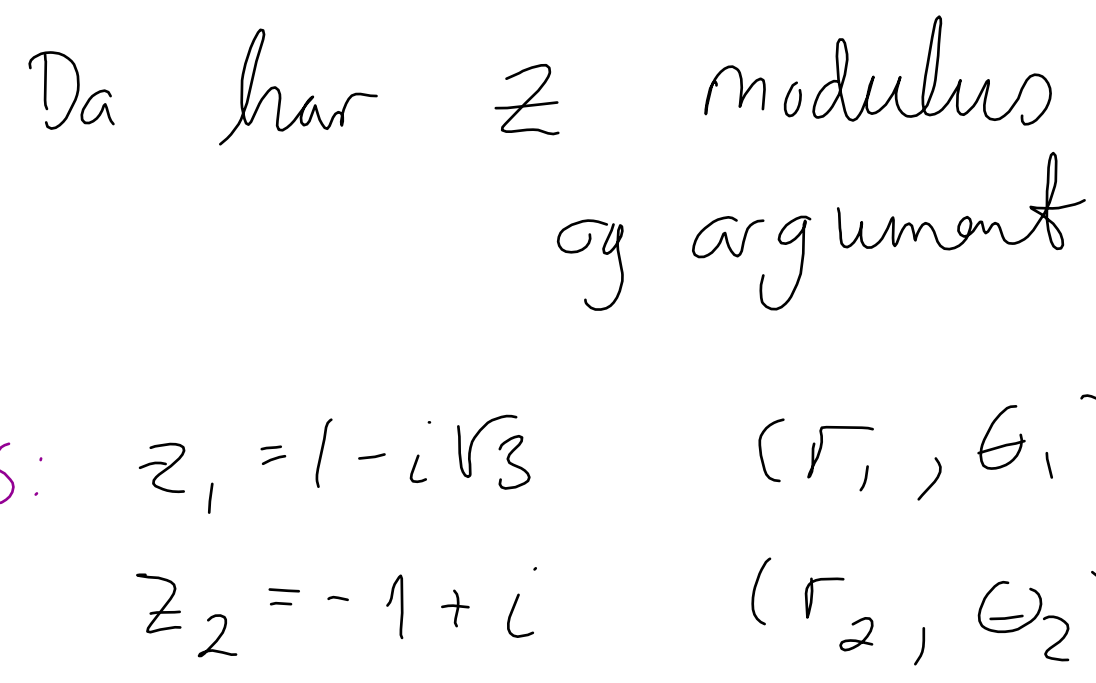
$z_2 = -1 + i$ $\quad (r_2, \theta_2) = (\sqrt{2}, \frac{3\pi}{4})$

$r_1 \cdot r_2 = 2 \cdot \sqrt{2}$

$$\theta_1 + \theta_2 = \frac{5\pi}{3} + \frac{3\pi}{4} = \frac{20\pi + 9\pi}{12} = \frac{29\pi}{12} = \frac{5\pi}{12} + \frac{24\pi}{12}$$

2π

$$z = 2\sqrt{2}\left(\cos\frac{5\pi}{12} + i\sin\frac{5\pi}{12}\right)$$



BEVIS

$$a_1 \equiv r_1 \cos\theta_1 \qquad a_2 = r_2 \cos\theta_2$$

$$b_1 = r_1 \sin\theta_1 \qquad b_2 = r_2 \sin\theta_2$$

$$z_1 \cdot z_2 = (a_1 a_2 - b_1 b_2) + i(a_1 b_2 + a_2 b_1)$$

$$= (r_1 \cos\theta_1 \cdot r_2 \cos\theta_2 - r_1 \sin\theta_1 r_2 \sin\theta_2)$$

$$+ i(r_1 \cos\theta_1 \cdot r_2 \sin\theta_2 + r_2 \cos\theta_2 r_1 \sin\theta_1)$$

$$= r_1 r_2 \left[(\cos\theta_1 \cdot \cos\theta_2 - \sin\theta_1 \sin\theta_2) + i(\cos\theta_1 \sin\theta_2 + \cos\theta_2 \sin\theta_1) \right]$$

Ved summeformler for $\cos(\theta_1 + \theta_2)$ og $\sin(\theta_1 + \theta_2)$

$$= r_1 r_2 \left[\cos(\theta_1 + \theta_2) + i \sin(\theta_1 + \theta_2) \right]$$

QED

KOROLLAR: For $z_2 \neq 0$ har vi

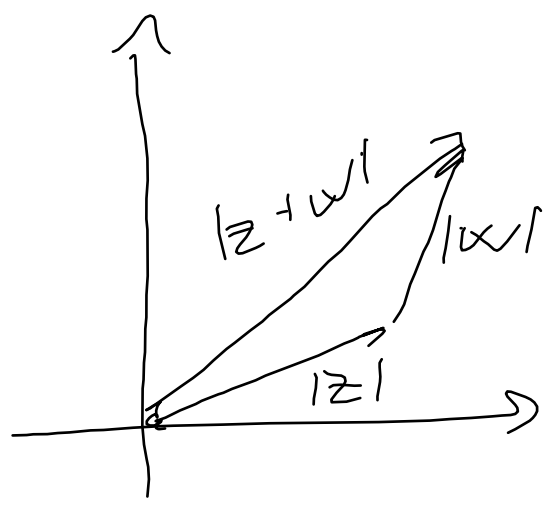
$$\frac{z_1}{z_2} = \frac{r_1}{r_2} \left[\cos(\theta_1 - \theta_2) + i \sin(\theta_1 - \theta_2) \right]$$

HERE GEOMETRISKE TOLKNINGER

SETNING 3.2.7

For $z, w \in \mathbb{C}$ gjelder

$$|z+w| \leq |z| + |w|$$



oppg. 3.2.22

OBS:
- $|z-w|$ er avstanden mellom z og w
- $|z-1|$ — " — z og 1
- $|z+i| = |z-(-i)|$ — " — z og $-i$

EKS: $\{z \in \mathbb{C} : |z+i| \leq 2\}$

