MAT 1100

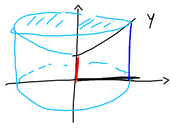
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MAT 1100 - 2023

(3) Volum - omdreiningslegeme om y-aksen

Område: · Mellom x-aksen og $y = \frac{1}{\sqrt{1-x^4}}$

Skisse



· Mellom $x = 0$ og $x = \frac{1}{\sqrt{2}}$

$$V = 2\pi \int_0^{\frac{1}{\sqrt{2}}} x \cdot \frac{1}{\sqrt{1-x^4}}\, dx$$

SUBST.
$u = x^2$
$du = 2x\,dx$

$$= \pi \int_{x=0}^{x=\frac{1}{\sqrt{2}}} \frac{1}{\sqrt{1-u^2}}\, du$$

$$= \pi \left[\arcsin(x^2) \right]_0^{\frac{1}{\sqrt{2}}}$$

$$= \pi \left[\arcsin\left(\frac{1}{2}\right) - \arcsin 0 \right]$$

$$= \pi \left[\frac{\pi}{6} - 0 \right]$$

$$= \frac{\pi^2}{6}$$

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(4) a) $\int \frac{4x+5}{x^2+4x+5}\,dx$

SUBST.
$u = x^2+4x+5$
$du = (2x+4)\,dx$

$+0$

$= \int \frac{2(2x+4) - 8 + 5}{x^2+4x+5}$

$= 2\int \frac{2x+4}{x^2+4x+5}\,dx - 3\int \frac{1}{x^2+4x+5}\,dx$

(I) (II)

(I) $2\int \frac{1}{u}\,du = 2\ln|x^2+4x+5| + C_1$

(II) $-3\int \frac{1}{x^2+4x+5}\,dx$

$= -3\int \frac{1}{(x+2)^2+1}\,dx$

$= -3\int \frac{1}{s^2+1}\,ds$

$= -3\arctan(x+2) + C_2$

- $b^2 - 4ac = 4^2 - 4\cdot 1\cdot 5 < 0$
- u^2+1

$+0$

$x^2 + 2\cdot x\cdot \frac{4}{2} + 2^2 - 2^2 + 5$

$(x+2)^2 + 1$

- SUBST:
  $s = x+2$
  $ds = dx$

(I) + (II) $= 2\ln(x^2+4x+5) - 3\arctan(x+2) + C$

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(4) b) $I = \int 2x \arctan(x+2)\,dx$

DELVIS:

$u = \arctan(x+2)$ $\quad u' = \frac{1}{(x+2)^2+1}\,[x+2]'$ (1 " ' ")

$v' = 2x$ $\quad v = x^2$

$I = x^2\arctan(x+2) - \int \frac{x^2}{(x+2)^2+1}\,dx$ samme grad

Rydder: $\frac{x^2}{(x+2)^2+1} = \frac{x^2}{x^2+4x+4+1} = \frac{x^2}{x^2+4x+5}$

- $x^2 : x^2+4x+5 = 1 - \frac{4x+5}{x^2+4x+5}$

$-(x^2+4x+5)$

$0 - 4x - 5$

$I = x^2\arctan(x+2) - \int 1 - \frac{4x+5}{x^2+4x+5}\,dx$

$= x^2\arctan(x+2) - x + \int \frac{4x+5}{x^2+4x+5}\,dx$ a)

$= x^2\arctan(x+2) - x + 2\ln(x^2+4x+5) - 3\arctan(x+2) + C$

$= (x^2-3)\arctan(x+2) - x + 2\ln(x^2+4x+5) + C$

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(4) La $f: \mathbb{R} \to \mathbb{R}$ være

$$f(x) = \begin{cases} 1 + \sin\frac{1}{x} \cdot \sin x & \text{for } x \neq 0 \\ 1 & \text{for } x = 0. \end{cases}$$

a) Skal vise at f er kontinuerlig i $x = 0$.
Nok å vise at $\lim_{x \to 0} f(x) = f(0) = 1$

$$\lim_{x \to 0} f(x) = \lim_{x \to 0} \left(1 + \underbrace{\sin\frac{1}{x}}_{\in [-1,1]} \cdot \underbrace{\sin x}_{0}\right)$$
$$= 1 + 0$$
$$= \underline{\underline{1}}, \text{ som var det jeg skulle vise.}$$

b) Skal vise at f ikke er deriverbar i $x = 0$

Ser på

$$f'(0) = \lim_{x \to 0} \frac{f(x) - f(0)}{x - 0}$$
$$= \lim_{x \to 0} \frac{1 + \sin\frac{1}{x} \cdot \sin x - 1}{x}$$
$$= \lim_{x \to 0} \underbrace{\sin\frac{1}{x}}_{[-1,1]} \cdot \underbrace{\frac{\sin x}{x}}_{1}$$

Grenseverdien vil anta alle verdier mellom $[-1, 1]$ vilkårlig nær $x = 0$ og $f'(0)$ eksisterer ikke.
Så f er ikke deriverbar i $x = 0$.

MAT 1100 - 2019

(5) Skal avgjøre om $\int_1^\infty \frac{\arctan x}{x^2}\,dx$ konvergerer eller divergerer.

- Vi vet at $\int_1^\infty \frac{1}{x^2}\,dx$ konvergerer
- På $[1,\infty)$ er $\frac{\arctan x}{x^2}$ positiv og kontinuerlig
- Kan bruke g.s.k.

$$\lim_{x\to\infty} \frac{\frac{\arctan x}{x^2}}{\frac{1}{x^2}} = \lim_{x\to\infty} \arctan x = \frac{\pi}{2} < \infty$$

- Så ved g.s.k. konvergerer integralet.

MAT-INF-2022

(7) Løs $y'' - y' - 6y = \underbrace{3x - \frac{3}{2}}$ med $y(0) = \frac{1}{3}$, $y'(0) = 2$

2. ordens diff.likn. — inhomogen — Nøyaktig én løsning

$y = y_h + y_p$

y_h: Kar.pol: $r^2 - r - 6 = 0$

$$r = \frac{-(-1) \pm \sqrt{(-1)^2 - 4 \cdot 1 \cdot (-6)}}{2 \cdot 1}$$

$$r = \frac{1 \pm 5}{2}$$

$r_1 = -2$ $\quad r_2 = 3$

$$y_h(x) = Ce^{-2x} + De^{3x}, \quad C, D \in \mathbb{R}$$

y_p: $y_p = Ax + B$, $A, B \in \mathbb{R}$

$y_p' = A$

$y_p'' = 0$

$y_p'' - y_p' - 6y_p = 3x - \frac{3}{2}$

$0 - A - 6(Ax + B) = 3x - \frac{3}{2}$

$-A - 6B - 6Ax = 3x - \frac{3}{2}$

- $-6A = 3$

$A = -\frac{1}{2}$

- $-A - 6B = -\frac{3}{2}$

$-(-\frac{1}{2}) - 6B = -\frac{3}{2}$

$-6B = -2$

$B = \frac{1}{3}$

$$y_p(x) = -\frac{1}{2}x + \frac{1}{3}$$

TOTALT: $y(x) = Ce^{-2x} + De^{3x} - \frac{1}{2}x + \frac{1}{3}$, $C, D \in \mathbb{R}$

Initialbetingelser:

- $y(0) = \frac{1}{3}$
- $y'(0) = 2$

- $y(0) = Ce^0 + De^0 - \frac{1}{2} \cdot 0 + \frac{1}{3}$

$= C + D + \frac{1}{3} = \frac{1}{3}$

- $C + D = 0$

- $y'(x) = C(-2)e^{-2x} + D \cdot 3e^{3x} - \frac{1}{2}$

$y'(0) = -2C + 3D - \frac{1}{2} = 2$

- $-2C + 3D = \frac{5}{2}$

- $C = -D$ $\qquad 2D + 3D = \frac{5}{2}$

$5D = \frac{5}{2}$

$D = \frac{1}{2}$

$C = -\frac{1}{2}$

$$y(x) = -\frac{1}{2}e^{-2x} + \frac{1}{2}e^{3x} - \frac{1}{2}x + \frac{1}{3}$$

SJEKK AT DENNE VIRKELIG ER LØSNINGEN!