Løsninger til oppgaver til uke 37 (ved Arne Hole)

Oppgave 1:

Oppgave 2:

Oppgave 3:

(b) $\sqrt{2}$ på begge

(c) $3\pi/4$ og $5\pi/4$

(d) $z = \sqrt{2}e^{i(3\pi/4)}$, $w = \sqrt{2}e^{i(5\pi/4)}$

Oppgave 4:

(b) $\mathfrak{Re}\, z = -1$, $\mathfrak{Im}\, z = -\sqrt{3}$, $z = (-1) + (-\sqrt{3})i$

Oppgave 5:

$e^{i(\pi/4)} = (1/\sqrt{2}) + (1/\sqrt{2})i$, $e^{i(\pi/3)} = \frac{1}{2} + \frac{\sqrt{3}}{2}i$, $e^{i(2\pi/3)} = -\frac{1}{2} + \frac{\sqrt{3}}{2}i$, $e^{i\pi} = -1$, $\frac{1}{2}e^{i(3\pi/2)} = -\frac{1}{2}i$, $2e^{i(\pi/3)} = 1 + \sqrt{3}i$

Oppgave 6:

$z + w = 9 + i$, $zw = 26 + 22i$

Oppgave 7:

$2e^{i(-\pi/3)} = 1 - \sqrt{3}i$, $5e^{i\cdot 1} = 5\cos 1 + (5\sin 1)i \approx 2.7 + (4.2)i$

Oppgave 8:

$-2 + (-2)i = \sqrt{8}e^{i(5\pi/4)}$, og $3 + 4i = 5e^{i\theta}$, der $\theta = \arccos(3/5) \approx 0.927$

Oppgave 9:

Oppgave 10:

(a) $6 - 2i$

(b) $-3 - 4i$

(c) $1 + 3i$

(d) $3 + 8i$

(e) $4 + 12i$

(f) $2 + 4i$

(g) 2

(h) $-19 - 13i$

(i) $\frac{11}{10} - \frac{1}{5}i$

(j) $-i$

(k) $\frac{4}{25} + \frac{3}{25}i$

(l) $-5i$

(m) $-2 + 2i$

(n) i

(o) $-1 - i$

(p) $-(7/625) - (24/625)i$

(q) $1 + 4i$

Oppgave 11:

(a) $\overline{4 + 3i} = 4 - 3i$, $\overline{-2 - 2i} = -2 + 2i$

(b) $z = re^{i\theta}$ gir $\overline{z} = re^{i(-\theta)} = re^{-i\theta}$. Så konjugasjon gir speiling om den reelle aksen.

(c)

Oppgave 12:

Den andre kvadratroten er $-2 - 3i$

Oppgave 13:

(a) $3i$

(b) i

(c) $4i$

(d) $i/2$

(e) $(1/\sqrt{2}) + (1/\sqrt{2})i$

(f) $8^{1/4}\cos(\pi/8) + 8^{1/4}\sin(\pi/8) \approx 1.55 + (0.64)i$

Oppgave 14:

(a) $w_0 = 2^{1/6}e^{i(\pi/12)}$, $w_1 = 2^{1/6}e^{i(3\pi/4)}$, $w_2 = 2^{1/6}e^{i(17\pi/12)}$

(b) $w_0 = 2e^{i(\pi/3)}$, $w_1 = -2$, $w_2 = 2e^{i(5\pi/3)}$

(c) $w_0 = e^{i(\pi/6)}$, $w_1 = e^{i(5\pi/6)}$, $w_2 = e^{i(3\pi/2)}$

Oppgave 15:

$w_0 = 1$, $w_1 = i$, $w_2 = -1$, $w_3 = -i$

Oppgave 16:

$z_1 = (\sqrt{3}/2) + (1/2)i$, $z_2 = -(\sqrt{3}/2) + (1/2)i$, $z_3 = -i$

Oppgave 17:

$z = -1$ eneste løsning

Oppgave 18:

$p \pm qi$

Oppgave 19:

(a) $5 \pm 3i$

(b) $\pm 2i$

(c) $-1 \pm 7i$

(d) $3 \pm i$

(e) 9 og -3

(f) $-7 \pm 2i$

Oppgave 20:

(a) $-\frac{1}{2} \pm \frac{\sqrt{3}}{2}i$

(b) $-\frac{1}{2}i \pm \frac{3}{2}$

(c) 0 og $-2 \pm 2i$

(d) ± 1, $\pm i$, $\pm(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}i)$, $\pm(-\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}i)$

Oppgave 21:

$(\sqrt{2} - 2) + (\sqrt{2} - 2)i$ og $(-\sqrt{2} - 2) + (-\sqrt{2} - 2)i$

Oppgave 22:

Hint: Skriv det komplekse tallet $e^{i\pi}$ på rektangulær form.

Oppgave 23:

Hint: $\cos\theta + i\sin\theta = e^{i\theta}$.