

Repetisjons forelesning 20/11

Ekamen mat 1110, avsluttende, 2024

$$1) A = \begin{pmatrix} 0.7 & 0 & 0.3 \\ 0 & 0.5 & 0 \\ 0.6 & 0 & 0.4 \end{pmatrix}$$

a) Egenverdier/vektorer:

$$\begin{aligned} \det(\lambda I - A) &= \begin{vmatrix} \lambda - 0.7 & 0 & -0.3 \\ 0 & \lambda - 0.5 & 0 \\ -0.6 & 0 & \lambda - 0.4 \end{vmatrix} = (\lambda - 0.5) \begin{vmatrix} \lambda - 0.7 & -0.3 \\ -0.6 & \lambda - 0.4 \end{vmatrix} \\ &= (\lambda - 0.5) ((\lambda - 0.7)(\lambda - 0.4) - 0.18) \\ &= (\lambda - 0.5) (\lambda^2 - 1.1\lambda + 0.28 - 0.18) \\ &= (\lambda - 0.5) (\lambda^2 - 1.1\lambda + 0.1) \quad \lambda = \frac{1.1 \pm \sqrt{1.21 - 0.4}}{2} \\ &\quad \lambda = 0.5 \quad \quad \quad = \frac{1.1 \pm 0.9}{2} \Rightarrow \lambda = 1, \lambda = 0.1 \\ &= (\lambda - 0.5)(\lambda - 0.1)(\lambda - 1) \end{aligned}$$

egenverdier: $\lambda_1 = 0.1$, $\lambda_2 = 0.5$, $\lambda_3 = 1$.

egenvektorer: $\lambda_1 = 0.1$

$$\lambda_1 I - A = \begin{pmatrix} -0.6 & 0 & -0.3 \\ 0 & -0.4 & 0 \\ -0.6 & 0 & -0.3 \end{pmatrix} \sim \begin{pmatrix} 1 & 0 & \frac{1}{2} \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \begin{aligned} x_1 + \frac{1}{2}x_3 &= 0 \\ x_2 &= 0 \end{aligned}$$

egenvektor: $\begin{pmatrix} -\frac{1}{2}x_3 \\ 0 \\ x_3 \end{pmatrix}$ spesiell egenvektor: $\vec{v}_1 = \begin{pmatrix} -1 \\ 0 \\ 2 \end{pmatrix}$ ($x_3 = 2$)

egenvektorer $\lambda_2 = 0.5$

$$\begin{aligned} \lambda_2 I - A &= \begin{pmatrix} -0.2 & 0 & -0.3 \\ 0 & 0 & 0 \\ -0.6 & 0 & 0.1 \end{pmatrix} \sim \begin{pmatrix} 1 & 0 & \frac{3}{2} \\ -0.6 & 0 & 0.1 \\ 0 & 0 & 0 \end{pmatrix} \sim \begin{pmatrix} 1 & 0 & \frac{3}{2} \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix} \\ &\sim \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix} \quad \begin{aligned} x_1 &= 0 \\ x_3 &= 0 \end{aligned} \Rightarrow \vec{v}_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \text{ spesiell egenvektor} \end{aligned}$$

eigenvektor $\lambda_3 = 1$

$$\lambda_3 I - A = \begin{pmatrix} 0.3 & 0 & -0.3 \\ 0 & 0.5 & 0 \\ -0.6 & 0 & 0.6 \end{pmatrix} \sim \begin{pmatrix} 1 & 0 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{matrix} x_1 - x_3 = 0 \\ x_2 = 0 \end{matrix}$$

$$\Rightarrow \begin{pmatrix} x_3 \\ 0 \\ x_3 \end{pmatrix} \text{ egenvektor} \Rightarrow \vec{v}_3 = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} \text{ spesiell egenr.}$$

b) $\vec{x}_0 = \begin{pmatrix} 2 \\ 3 \\ 8 \end{pmatrix}$

$$\begin{pmatrix} \vec{v}_1 & \vec{v}_2 & \vec{v}_3 & \vec{x}_0 \\ -1 & 0 & 1 & 2 \\ 0 & 1 & 0 & 3 \\ 2 & 0 & 1 & 8 \end{pmatrix} \sim \begin{pmatrix} 1 & 0 & -1 & -2 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 3 & 12 \end{pmatrix} \sim \begin{pmatrix} 1 & 0 & -1 & -2 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 4 \end{pmatrix}$$

$$\sim \begin{pmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 4 \end{pmatrix}$$

$$\vec{x}_0 \Downarrow = 2\vec{v}_1 + 3\vec{v}_2 + 4\vec{v}_3$$

$$\lim_{n \rightarrow \infty} A^n \vec{x}_0 = \lim_{n \rightarrow \infty} (2 A^n \vec{v}_1 + 3 A^n \vec{v}_2 + 4 A^n \vec{v}_3)$$

$$= \lim_{n \rightarrow \infty} (2 \cdot 0.1^n \vec{v}_1 + 3 \cdot 0.5^n \vec{v}_2 + 4 \cdot 1^n \vec{v}_3)$$

$$= \lim_{n \rightarrow \infty} 4 \vec{v}_3 = 4 \vec{v}_3 = 4 \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 4 \\ 0 \\ 4 \end{pmatrix}$$

Eksamen mat 110 midtveis 2024

10. $\vec{v}_1 = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}, \vec{v}_2 = \begin{pmatrix} 3 \\ 4 \\ 2 \end{pmatrix}$

$$\begin{pmatrix} 1 & 3 & 1 \\ 2 & 4 & 1 \\ 1 & 2 & 0 \end{pmatrix} \sim \begin{pmatrix} 1 & 3 & 1 \\ 0 & -2 & -1 \\ 0 & 1 & -1 \end{pmatrix} \sim \begin{pmatrix} 1 & 3 & 1 \\ 0 & 1 & -1 \\ 0 & 0 & -4 \end{pmatrix} \sim \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{pmatrix}$$

ser: siste søyle pivotsøyle $\Rightarrow \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix}$ kan ikke uttrykkes ved $\vec{v}_1, \vec{v}_2$.

Eksamen mat-inf 1100, avsluttende 2023

3. $f(x) = x^3 - x - 1$, p_2 interpolerer i 1, 2, 3.

$$p_2(x) = c_0 + c_1(x-1) + c_2(x-1)(x-2)$$

$$f(1) = -1 = c_0 \Rightarrow c_0 = -1$$

$$f(2) = 5 = c_0 + c_1 \Rightarrow c_1 = 6$$

$$f(3) = 23 = c_0 + 2c_1 + 2c_2 \Rightarrow c_2 = \frac{23 + 1 - 12}{2} = 6$$

$$\begin{aligned} p_2(x) &= -1 + 6(x-1) + 6(x-1)(x-2) = -1 + 6x - 6 + 6x^2 - 18x + 12 \\ &= 6x^2 - 12x + 5 \end{aligned}$$

$$\begin{aligned} \int_1^3 (6x^2 - 12x + 5) dx &= \left[2x^3 - 6x^2 + 5x \right]_1^3 \\ &= 2(27 - 1) - 6(9 - 1) + 5(3 - 1) \\ &= 52 - 48 + 10 = 14. \end{aligned}$$

$$\begin{aligned} \text{p\aa Newtonform: } \int_1^3 (x-1)(x-2) dx &= \left[\frac{1}{2}(x-1)^2(x-2) \right]_1^3 - \int_1^3 \frac{1}{2}(x-1)^2 dx \\ &= \dots \end{aligned}$$

evt. med Simpsons metode:

$$\begin{aligned} \int_1^3 p_2(x) dx &= \frac{h}{3} (f(1) + 4f(2) + f(3)) \\ &= \frac{1}{3} (-1 + 4 \cdot 5 + 23) \\ &= \frac{1}{3} (-1 + 20 + 23) = \frac{42}{3} = 14 \end{aligned}$$

5. Midtpunktsmetoden p\aa $\int_0^2 \sin\left(\frac{\pi}{2}x + \frac{\pi}{8}\right) dx$, 4 delintervaller

$$h = \frac{2-0}{4} = \frac{1}{2}$$

intervallene: $[0, \frac{1}{2}]$, $[\frac{1}{2}, 1]$, $[1, \frac{3}{2}]$, $[\frac{3}{2}, 2]$

midtpunkterne: $\frac{1}{4}$ $\frac{3}{4}$ $\frac{5}{4}$ $\frac{7}{4}$

metoden gir $h \left(f\left(\frac{1}{4}\right) + f\left(\frac{3}{4}\right) + f\left(\frac{5}{4}\right) + f\left(\frac{7}{4}\right) \right)$

$$\begin{array}{cccc} \sin\left(\frac{\pi}{8} + \frac{\pi}{8}\right) & \sin\left(\frac{3\pi}{8} + \frac{\pi}{8}\right) & \sin\left(\frac{5\pi}{8} + \frac{\pi}{8}\right) & \sin\left(\frac{7\pi}{8} + \frac{\pi}{8}\right) \\ \parallel & \parallel & \parallel & \parallel \\ \sin\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2} & \sin\left(\frac{\pi}{2}\right) = 1 & \sin\left(\frac{6\pi}{8}\right) = \frac{\sqrt{2}}{2} & \sin(\pi) = 0 \end{array}$$

$$= \frac{1}{2} \left(\frac{\sqrt{2}}{2} + 1 + \frac{\sqrt{2}}{2} + 0 \right) = \frac{1}{2} (\sqrt{2} + 1).$$

del 2

$$3 \quad f'(a) \approx \frac{-4f(a-h) + 3f(a) + f(a+2h)}{6h}$$

Viser at er eksakt for $f(x) = 1$:

$$VS: f'(a) = \underline{0}$$

$$HS: \frac{-4 + 3 + 1}{6h} = \underline{0}$$

Viser at er eksakt for $f(x) = x$:

$$VS: f'(a) = 1$$

$$HS: \frac{-4(a-h) + 3a + (a+2h)}{6h} = \frac{4h + 2h}{6h} = \frac{6h}{6h} = 1.$$

Viser at er eksakt for $f(x) = x^2$:

$$VS: f'(a) = \underline{2a}$$

$$HS: \frac{-4(a-h)^2 + 3a^2 + (a+2h)^2}{6h}$$

$$= \frac{-4a^2 + 8ah - 4h^2 + 3a^2 + a^2 + 4ah + 4h^2}{6h}$$

$$= \frac{12ah}{6h} = \underline{2a}$$

Når formel eksakt for $f \Rightarrow$ også viktig for cf :

$$\begin{aligned}(cf)'(a) &= cf'(a) = c \frac{-4f(a-h) + 3f(a) + f(a+2h)}{6h} \\ &= \frac{-4(cf)(a-h) + 3(cf)(a) + (cf)(a+2h)}{6h}\end{aligned}$$

$\Rightarrow$ viktig også for cf .

Når formel eksakt for f og $g \Rightarrow$ også viktig for $f+g$:

$$\begin{aligned}(f+g)'(a) &= f'(a) + g'(a) = \frac{-4f(a-h) + 3f(a) + f(a+2h)}{6h} \\ &\quad + \frac{-4g(a-h) + 3g(a) + g(a+2h)}{6h} \\ &= \frac{-4(f+g)(a-h) + 3(f+g)(a) + (f+g)(a+2h)}{6h}\end{aligned}$$

$\Rightarrow$ viktig også for $f+g$.