

MAT 1110 midtveis, 2024

10.  $\vec{v}_1 = \begin{pmatrix} 1 \\ 2 \\ 1 \\ 1 \end{pmatrix}$ ,  $\vec{v}_2 = \begin{pmatrix} 3 \\ 4 \\ 2 \\ 0 \end{pmatrix}$

$$\begin{pmatrix} 1 & 3 & 1 \\ 2 & 4 & 1 \\ 1 & 2 & 0 \\ 1 & 0 & 2 \end{pmatrix} \sim \begin{pmatrix} 1 & 3 & 1 \\ 0 & -2 & -1 \\ 0 & -1 & -1 \\ 0 & -3 & 1 \end{pmatrix} \sim \begin{pmatrix} 1 & 3 & 1 \\ 0 & -2 & -1 \\ 0 & -3 & 1 \end{pmatrix}$$

$$\sim \begin{pmatrix} 1 & 3 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 4 \end{pmatrix} \sim \begin{pmatrix} 1 & 3 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}$$

Siste søyle er en pivotsøyle  $\Rightarrow$  ingen løsning.

11.  $\begin{pmatrix} 2 & 6 & 1 \\ 4 & 12 & 4 \\ 1 & 3 & 2 \end{pmatrix} \sim \begin{pmatrix} 1 & 3 & 2 \\ 4 & 12 & 4 \\ 2 & 6 & 1 \end{pmatrix} \sim \begin{pmatrix} 1 & 3 & 2 \\ 0 & 0 & -4 \\ 0 & 0 & -3 \end{pmatrix}$

$$\sim \begin{pmatrix} 1 & 3 & 2 \\ 0 & 0 & -4 \\ 0 & 0 & 0 \end{pmatrix}$$

søyle 1 og 3 er pivotsøylar.

12.  $\begin{vmatrix} 2 & 3 & -2 & 1 \\ 0 & 1 & 3 & 0 \\ 1 & -1 & 2 & 0 \\ 0 & 0 & 1 & 0 \end{vmatrix} = - \begin{vmatrix} 0 & 1 & 3 \\ 1 & -1 & 2 \\ 0 & 0 & 1 \end{vmatrix} = - \begin{vmatrix} 0 & 1 \\ 1 & -1 \end{vmatrix} = -(-1) = 1$

13.  $\begin{pmatrix} 1 & 1 & 2 \\ 0 & 2 & 0 \\ 2 & 1 & 1 \end{pmatrix}$ :  $\det(A - \lambda I) = \begin{vmatrix} 1-\lambda & 1 & 2 \\ 0 & 2-\lambda & 0 \\ 2 & 1 & 1-\lambda \end{vmatrix}$

$$= (2-\lambda) \begin{vmatrix} 1-\lambda & 2 \\ 2 & 1-\lambda \end{vmatrix} = (2-\lambda)((1-\lambda)^2 - 4)$$

$(1-\lambda)^2 - 4 = 0$  når  $(1-\lambda)^2 = 4 \Rightarrow 1-\lambda = \pm 2 \Rightarrow \lambda = 1 \pm 2$

eigenverdier:  $\lambda_1 = -1$ ,  $\lambda_2 = 3$ ,  $\lambda_3 = 2$

$$14. A = \begin{pmatrix} 0.8 & 0.7 \\ 0.2 & 0.3 \end{pmatrix}$$

$$\begin{aligned} \det(A - \lambda I) &= \begin{vmatrix} 0.8 - \lambda & 0.7 \\ 0.2 & 0.3 - \lambda \end{vmatrix} = (0.8 - \lambda)(0.3 - \lambda) - 0.14 \\ &= \lambda^2 - 1.1\lambda + 0.1 = 0 \quad \text{når} \\ \lambda &= \frac{1.1 \pm \sqrt{1.21 - 0.4}}{2} \\ &= \frac{1.1 \pm 0.9}{2} \end{aligned}$$

$$\lambda_1 = 0.1, \quad \lambda_2 = 1$$

Egenvektor  $\lambda_1 = 0.1$

$$A - \lambda_1 I = \begin{pmatrix} 0.7 & 0.7 \\ 0.2 & 0.2 \end{pmatrix} \sim \begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix} \Rightarrow x_1 + x_2 = 0 \Rightarrow x_1 = -x_2$$

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} -x_2 \\ x_2 \end{pmatrix} = x_2 \begin{pmatrix} -1 \\ 1 \end{pmatrix}$$

spec. egenvektor

Egenvektor  $\lambda_2 = 1$

$$A - \lambda_2 I = \begin{pmatrix} -0.2 & 0.7 \\ 0.2 & -0.7 \end{pmatrix} \sim \begin{pmatrix} 1 & -\frac{7}{2} \\ 0 & 0 \end{pmatrix} \Rightarrow x_1 - \frac{7}{2}x_2 = 0$$
$$\Downarrow$$
$$x_1 = \frac{7}{2}x_2$$

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} \frac{7}{2}x_2 \\ x_2 \end{pmatrix} = x_2 \begin{pmatrix} \frac{7}{2} \\ 1 \end{pmatrix}$$

$\begin{pmatrix} 7 \\ 2 \end{pmatrix}$  spesiell egenvektor.

$$15) \quad x + 3y + 2z = 2$$

$$4x + 2y + z = 4$$

$$3x - y - z = a$$

$$\begin{pmatrix} 1 & 3 & 2 & 2 \\ 4 & 2 & 1 & 4 \\ 3 & -1 & -1 & a \end{pmatrix} \sim \begin{pmatrix} 1 & 3 & 2 & 2 \\ 0 & -10 & -7 & -4 \\ 0 & -10 & -7 & a-6 \end{pmatrix}$$

$$\sim \begin{pmatrix} 1 & 3 & 2 & 2 \\ 0 & -10 & -7 & -4 \\ 0 & 0 & 0 & a-2 \end{pmatrix}$$

har løsning kun når  $a-2=0$ , det vil si når  $a=2$

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4  $A = \begin{pmatrix} -1 & 2/3 \\ -4 & 7/3 \end{pmatrix}$

$$\det(A - \lambda I) = \begin{vmatrix} -1-\lambda & \frac{2}{3} \\ -4 & \frac{7}{3}-\lambda \end{vmatrix} = (-1-\lambda)\left(\frac{7}{3}-\lambda\right) + \frac{8}{3}$$

$$= \lambda^2 - \frac{4}{3}\lambda + \frac{1}{3} = 0$$

$$\lambda = \frac{\frac{4}{3} \pm \sqrt{\frac{16}{9} - \frac{4}{3}}}{2}$$

$$= \frac{\frac{4}{3} \pm \sqrt{\frac{4}{9}}}{2} = \frac{\frac{4}{3} \pm \frac{2}{3}}{2}$$

$$= \frac{2}{3} \pm \frac{1}{3}$$

$$\Rightarrow \lambda_1 = \frac{1}{3}, \lambda_2 = 1$$

egenvektor  $\lambda_1 = \frac{1}{3}$ :

$$A - \frac{1}{3}I = \begin{pmatrix} -\frac{4}{3} & \frac{2}{3} \\ -4 & 2 \end{pmatrix} \sim \begin{pmatrix} 1 & -\frac{1}{2} \\ 0 & 0 \end{pmatrix} \Rightarrow x_1 = \frac{1}{2}x_2$$

$$\Downarrow$$

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} \frac{1}{2}x_2 \\ x_2 \end{pmatrix} = x_2 \begin{pmatrix} \frac{1}{2} \\ 1 \end{pmatrix}$$

$\Rightarrow \begin{pmatrix} 1 \\ 2 \end{pmatrix}$  egenvektor.

eigenvektor  $\lambda_2 = 1$

$$A - I = \begin{pmatrix} -2 & \frac{2}{3} \\ -4 & \frac{4}{3} \end{pmatrix} \sim \begin{pmatrix} 1 & -\frac{1}{3} \\ 0 & 0 \end{pmatrix} \Rightarrow x_1 = \frac{1}{3} x_2$$

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} \frac{1}{3} x_2 \\ x_2 \end{pmatrix} = x_2 \begin{pmatrix} \frac{1}{3} \\ 1 \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} 1 \\ 3 \end{pmatrix} \text{ eigenvektor.}$$

5.  $\lim_{n \rightarrow \infty} A^n (a \vec{v}_1 + b \vec{v}_2) = \lim_{n \rightarrow \infty} a A^n \vec{v}_1 + \lim_{n \rightarrow \infty} b A^n \vec{v}_2$

$$= \lim_{n \rightarrow \infty} a 1^n \vec{v}_1 + \lim_{n \rightarrow \infty} b \left(\frac{1}{3}\right)^n \vec{v}_2$$

$$= a \vec{v}_1$$

14.  $A = \begin{pmatrix} 1 & 1 & 2 \\ 1 & -2 & 3 \\ 1 & 4 & 1 \end{pmatrix} \sim \begin{pmatrix} 1 & 1 & 2 \\ 0 & -3 & 1 \\ 0 & 3 & -1 \end{pmatrix} \sim \begin{pmatrix} 1 & 1 & 2 \\ 0 & -3 & 1 \\ 0 & 0 & 0 \end{pmatrix}$

$\Rightarrow A$  ikke inverterbar.  $\Leftrightarrow Ax = \vec{0}$  har uendelig mange løsninger.

15.

$$\begin{aligned} x + y + 2z &= b_1 \\ x - 2y + 3z &= b_2 \\ x + 4y + z &= b_3 \end{aligned}$$

$$\begin{pmatrix} 1 & 1 & 2 & b_1 \\ 1 & -2 & 3 & b_2 \\ 1 & 4 & 1 & b_3 \end{pmatrix} \sim \begin{pmatrix} 1 & 1 & 2 & b_1 \\ 0 & -3 & 1 & b_2 - b_1 \\ 0 & 3 & -1 & b_3 - b_1 \end{pmatrix}$$

$$\sim \begin{pmatrix} 1 & 1 & 2 & b_1 \\ 0 & -3 & 1 & b_2 - b_1 \\ 0 & 0 & 0 & b_2 + b_3 - 2b_1 \end{pmatrix}$$

Ingen løsning når  $b_2 + b_3 - 2b_1 \neq 0$

# Eksamen mat 1100 2020, avsluttende

## Oppgave 7:

$$\left. \begin{array}{l} X_n : \text{nyspirte pl.} \\ Y_n : \text{unge pl.} \\ Z_n : \text{voksne pl.} \end{array} \right\} \text{ etter } n \text{ år}$$

Opplysning 1:  $X_{n+1} = Y_n + Z_n$

Opplysning 2:  $Y_{n+1} = X_n$

Opplysning 3:  $Z_{n+1} = \underbrace{\frac{4}{5}Y_n}_{\text{avungpl.}} + \underbrace{\frac{3}{5}Z_n}_{\text{av voksne pl.}}$

$$\begin{pmatrix} X_{n+1} \\ Y_{n+1} \\ Z_{n+1} \end{pmatrix} = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & \frac{4}{5} & \frac{3}{5} \end{pmatrix} \begin{pmatrix} X_n \\ Y_n \\ Z_n \end{pmatrix}$$

b) er  $M = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & \frac{4}{5} & \frac{3}{5} \end{pmatrix}$  invertierbar?

$$\det M = - \begin{vmatrix} 1 & 1 \\ \frac{4}{5} & \frac{3}{5} \end{vmatrix} = - \left( \frac{3}{5} - \frac{4}{5} \right) \neq 0$$

Derfor er  $M$  invertierbar, og vi kan sette  $N = M^{-1}$ .

c) Ungplante gir frø til 2 nyspirte,  
halvparten av unge overlever, ellers samme opplysninger

$$\begin{array}{lcl} X_{n+1} & = & \underline{\underline{2Y_n + Z_n}} \\ Y_{n+1} & = & X_n \\ Z_{n+1} & = & \underline{\underline{\frac{1}{2}Y_n + \frac{3}{5}Z_n}} \end{array} \Rightarrow M = \begin{pmatrix} 0 & 2 & 1 \\ 1 & 0 & 0 \\ 0 & \frac{1}{2} & \frac{3}{5} \end{pmatrix}$$

