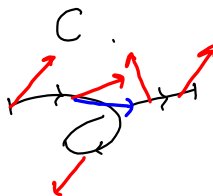


$$\vec{r}: [0,1] \rightarrow \mathbb{R}^n$$

$$f: \mathbb{R}^n \rightarrow \mathbb{R}$$

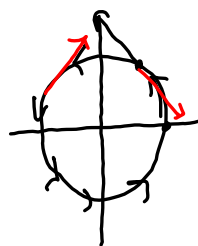


$$\int_C f ds := \int_0^1 f(\vec{r}(t)) \cdot \underset{\substack{\uparrow \\ |\vec{r}'(t)|}}{v(t)} dt$$

Vektorfelder:  $F: \mathbb{R}^n \rightarrow \mathbb{R}^n$ .

$$\int_C F \cdot dr = \int_0^1 F(\vec{r}(t)) \cdot \vec{r}'(t) dt$$

Eks:  $\text{La } \vec{r}(t) = (\cos(t), \sin(t)), t \in [0, 2\pi]$   
 $\vec{r}'(t) = (-\sin(t), \cos(t))$



$\text{La } F(x,y) = (y, -x)$

$$\int_C F \cdot dr = \int_0^{2\pi} F(\vec{r}(t)) \cdot \vec{r}'(t) dt$$

$$= \int_0^{2\pi} (\sin(t), -\cos(t)) \cdot (-\sin(t), \cos(t)) dt$$

$$= \int_0^{2\pi} -\sin^2 t - \cos^2 t dt = -2\pi$$

Andre weien:  $\vec{r}(t) = (\cos(-t), \sin(-t))$

$$\int_C F \cdot dr = \int_0^{2\pi} (\sin(-t), \cos(-t)) \cdot (\sin(-t), -\cos(-t)) dt$$

$$= \int_0^{2\pi} \sin^2(-t) + \cos^2(-t) dt$$

$$= 2\pi$$

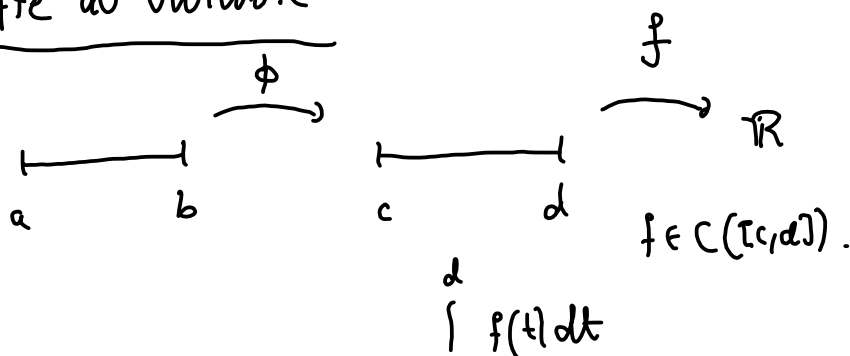
Dessom  $\vec{r}_1$  og  $\vec{r}_2$  er to ekvivalente  
 parametriseringer av kurven  $C$ ,  
 $\vec{r}_1(t) = \vec{r}_2(\phi(t))$ , og dessom  
 $\vec{F}$  er et vektorfelt har vi

(i) Dessom  $\vec{r}_1$  og  $\vec{r}_2$  har samme  
 orientering så har vi

$$\int_C \vec{F} \cdot d\vec{r}_1 = \int_C \vec{F} \cdot d\vec{r}_2, \text{ og}$$

(ii) dessom  $\vec{r}_1$  og  $\vec{r}_2$  har motsatt  
 orientering så har vi

$$\int_C \vec{F} \cdot d\vec{r}_1 = - \int_C \vec{F} \cdot d\vec{r}_2.$$

Skifte av variable:

Kan se på  $f \circ \phi$  som nå er kont.  
på  $[a, b]$ .

Er det en sammenheng mellom  $\int_a^b (f \circ \phi)$  og  $\int_c^d f$ ?

Eks:

Diagram illustrating an example. A horizontal line represents the interval  $[0, 1]$  on the left, with points  $0$  and  $1$  marked. An arrow labeled  $\phi$  points to the right, where another horizontal line represents the interval  $[0, 2]$  on the  $t$ -axis, with points  $0$  and  $2$  marked. Above the  $t$ -axis, a function  $f$  is shown as a horizontal line at  $y=1$ , with an arrow pointing to the right towards the label  $f(t)=1$ . Below the  $t$ -axis, the integral  $\int_0^2 1 dt = 2$  is written.

Below the diagram, the integral  $\int_0^1 1 \cdot 2 dt$  is written.

Teorem: La  $[a, b]$  og  $[c, d]$  være intervaller, og la  $\phi: [a, b] \rightarrow [c, d]$  være deriverbar med  $\phi(a)=c$  og  $\phi(b)=d$ . Da

$$\int_c^d f(t) dt = \int_a^b f(\phi(t)) \cdot \phi'(t) dt.$$

Der som  $\phi(a)=d$  og  $\phi(b)=c$ ,

$$\int_c^d f(t) dt = - \int_a^b f(\phi(t)) \cdot \phi'(t) dt,$$

Bevis for  $\pm$  likhet for integrals:

$$\vec{r}_1(t) = \vec{r}_2(\phi(t)) \quad \begin{array}{l} \text{Anta } a=c=0 \\ \text{og } b=d=1. \end{array}$$

$$\begin{aligned} \int_C \vec{F} \cdot d\vec{r}_1 &= \int_0^1 \vec{F}(\vec{r}_1(t)) \cdot \vec{r}_1'(t) dt \\ &= \int_0^1 \vec{F}(\vec{r}_2(\phi(t))) \cdot (\vec{r}_2(\phi(t)))' dt \\ &= \int_0^1 \vec{F}(\vec{r}_2(\phi(t))) \cdot \vec{r}_2'(\phi(t)) \cdot \phi'(t) dt \\ &= \pm \int_0^1 \vec{F}(\vec{r}_2(t)) \cdot \vec{r}_2'(t) dt = \pm \int_C \vec{F} d\vec{r}_2. \end{aligned}$$

Skifte-av-variabel-formel

$$\int_0^1 f(\phi(t)) \cdot \phi'(t) dt = \int_0^1 f(t) dt$$