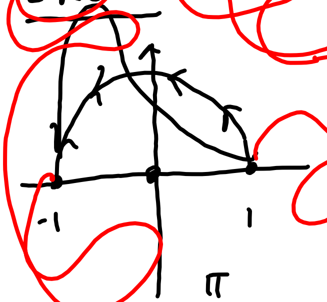
3.5 Gradienter og konservative felter.

Eks 1: $\vec{r}(t) = (\cos t, \sin t)$, $t \in [0, \pi]$.



$\vec{F}(x,y) = (y, x)$

$\int_C \vec{F} \cdot d\vec{r}$?

$$= \int_0^{\pi} \vec{F}(\vec{r}(t)) \cdot \vec{r}\,'(t)\, dt =$$

$$\int_0^{\pi} (\sin t, \cos t) \cdot (-\sin t, \cos t)\, dt$$

$$= \int_0^{\pi} -\sin^2 t + \cos^2 t \, dt$$

$$= \int_0^{\pi} \cos^2 t - (1 - \cos^2 t)\, dt$$

$$2{,}5e^{-17} = \int_0^{\pi} 2\cos^2 t - 1 \, dt.$$

MATLAB: quad('2*(cos(t)).^2 -1', 0, pi)

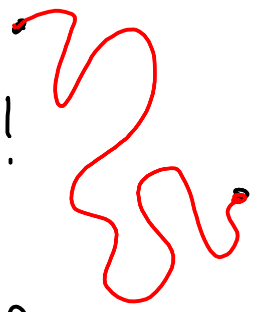
$= 0$.

Eks 2: Definer istedet: $\vec{r}(t) = (\cos(\pi t^2), \sin(\pi t^2))$, $t \in [0, 1]$.

$\int_C \vec{F} \cdot d\vec{r}$? $= 0$.

plot C i MATLAB!

$$\vec{F}(x,y) = (y, x).$$



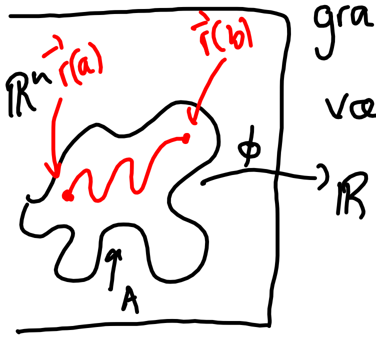
OBS: $\vec{F}$ e en gradient!

$$\phi(x,y) = x \cdot y$$

$$\vec{F} = \nabla\phi = \left(\frac{\partial\phi}{\partial x}, \frac{\partial\phi}{\partial y}\right).$$

En variabel: $\displaystyle\int_a^b f'(t)\,dt = f(b) - f(a).$

SETNING 3.5.1: Anta at $\phi : A \subset \mathbb{R}^n \longrightarrow \mathbb{R}$ er en funksjon med kontinuerlig gradient $\nabla\phi$, og la $\vec{r} : [a,b] \to A$ være en parametrisert kurve. Da har vi at



$$\int_C \nabla\phi \cdot d\vec{r} = \phi(\vec{r}(b)) - \phi(\vec{r}(a)).$$

Bevis: Husk: $\frac{d}{dt}[\phi(r(t))] = \nabla\phi(\vec{r}(t)) \cdot r'(t).$

$$\int_C \nabla\phi \cdot d\vec{r} = \int_a^b \nabla\phi(\vec{r}(t)) \cdot \vec{r}\,'(t)\,dt$$

$$= \int_a^b \frac{d}{dt}\left(\phi(r(t))\right) dt$$

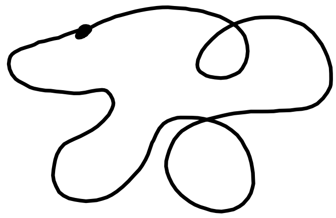
$$= \phi(\vec{r}(b)) - \phi(r(\vec{a})).$$

$\blacksquare$

DEF: Dersom F er et vektorfelt $F = \nabla\phi$ sies vi at F er <u>konservativt</u>.

ϕ kalles potensialet til F.

Observer: Dersom du integrerer et konservativt felt F over en lukket kurve blir utfallet null.

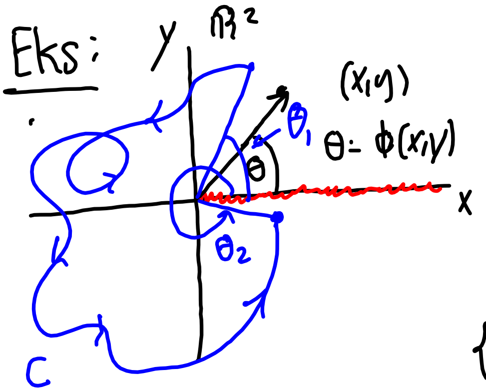


$\vec{r} : [a,b] \longrightarrow \mathbb{R}^n$

$\vec{r}(a) = \vec{r}(b)$

$$\int_C \vec{F} \cdot d\vec{r} = \phi(\vec{r}(b)) - \phi(\vec{r}(a)) = 0.$$

Eks:



Skal definere en funksjon ϕ på $\mathbb{R}^2 \setminus \{\mathbb{R}^+\}$.

$$= \{(x,y) \in \mathbb{R}^2 : x < 0 \text{ hvis } y = 0\}.$$

Definer $\phi(x,y)$ til å være vinkelen mellom den positive x-aksen og linja mellom origo og (x,y).

$$\int_C \nabla\phi \cdot dr = \theta_2 - \theta_1$$