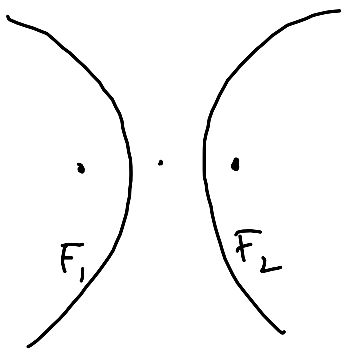
Hyperbler



Velg en $a>0$ s.a. $2a < |F_1F_2|$.

Hyperbelen med brennpunkter F_1 og F_2 og halakse a er mengden av punkter P s.a.

$$\big|\, |PF_1| - |PF_2| \,\big| = 2a .$$

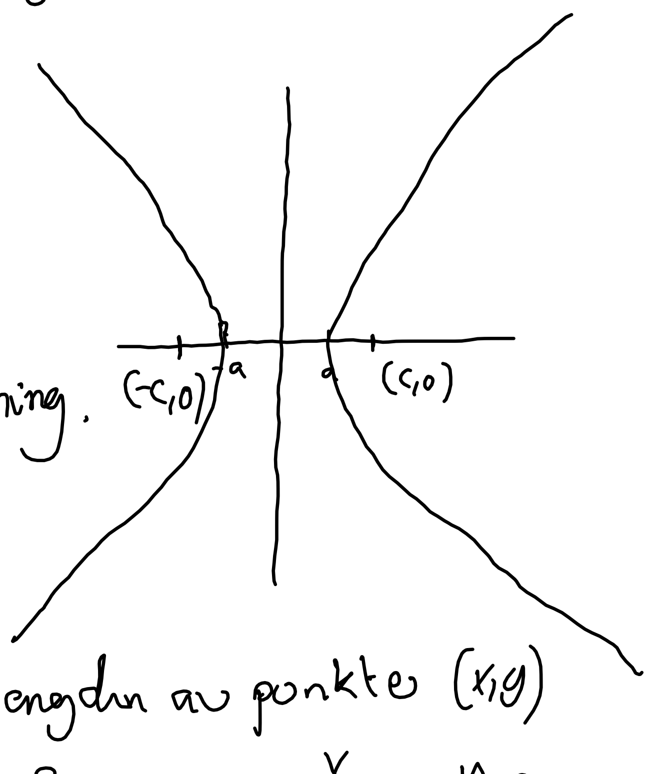
- Sett $F_1 = (c,0)$ og $F(-c,0)$ for en $c>0$.
- Velg $0<a<c$.
- Sett $b = \sqrt{c^2 - a^2}$

⋮

Stygg utregning.

⋮

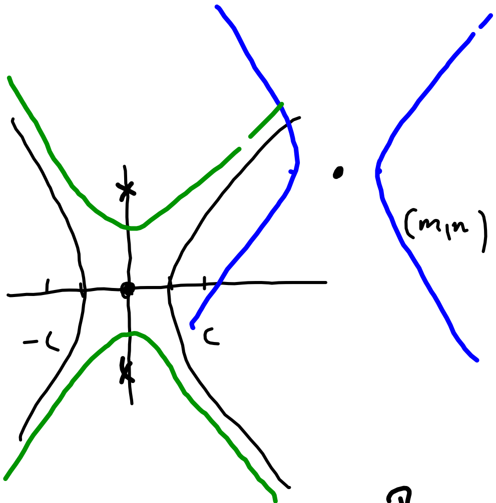
$\Downarrow$



Hyperbelen er mengden av punkter (x,y) som tilfredsstiller ligning $\left(\frac{x}{a}\right)^2 - \left(\frac{y}{b}\right)^2 = 1$.

SETNING 3.6.5: Ligningen $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ fremstiller en hyperbel med halvakse a, og brennpunkter $(c,0)$ og $(-c,0)$ der c er definert ved $c^2 = a^2 + b^2$.

$\boxed{\text{oppgave: Hva skjer hvis vi setter } 2a \geq |F_1F_2| \text{?}}$



Kan også sentrere en hyperbel i et punkt (m,n):

$$\left(\frac{x-m}{a}\right)^2 - \left(\frac{y-n}{b}\right)^2 = 1.$$

Brennpunkter: $(m-c,n)$, $(m+c,n)$.

Kan også bytte rollen til x og y:

$$\left(\frac{y}{b}\right)^2 - \left(\frac{x}{a}\right)^2 = 1.$$

- b er halvaksen.
- Brennpunkter: $(0,-c)$ og $(0,c)$.

EKS: $y^2 - 2y - 2x^2 - 3 = 0$.

Vis at ligningen fremstiller en hyperbel og finn brennpunkter, senter, og halvakse.

$$(y-1)^2 - 1 - 2x^2 - 3 = 0$$

$$(y-1)^2 - 2x^2 - 4 = 0.$$

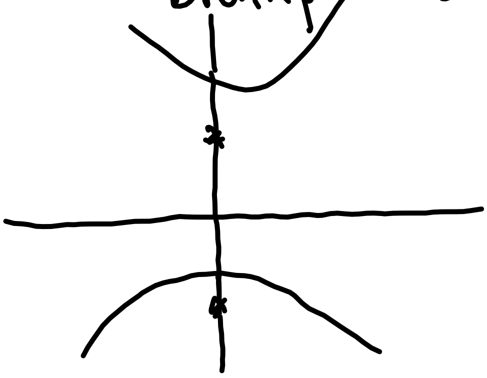
$$(y-1)^2 - 2x^2 = 4$$

$$\left(\frac{y-1}{2}\right)^2 - \left(\frac{x}{\sqrt{2}}\right)^2 = 1.$$

Senter: $(0, 1)$.

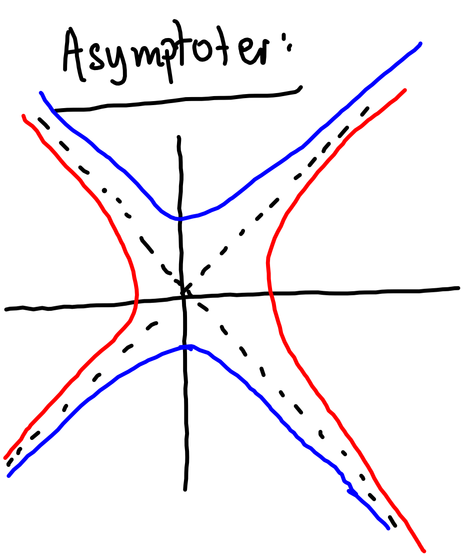
Halvakse: 2.

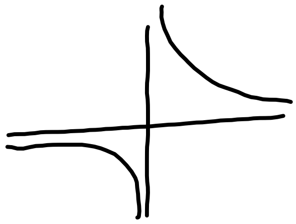
Brennpunkter: $c = \sqrt{a^2 + b^2} = \sqrt{2+4} = \sqrt{6}$.



$(0, 1+\sqrt{6})$, $(0, 1-\sqrt{6})$.

Asymptoter:





SETNING 3.6.6 Hyperblene $\left(\frac{x-m}{a}\right)^2-\left(\frac{y-n}{b}\right)^2=1$ og $\left(\frac{y-n}{b}\right)^2-\left(\frac{x-m}{a}\right)^2=1$ har begge asymptotene $(y-n)=\pm\frac{b}{a}(x-m)$.

"Bevis": Setter $m=n=0$.
Sjekker for $\left(\frac{x}{a}\right)^2-\left(\frac{y}{b}\right)^2=1$.

$$\frac{y^2}{b^2}=\frac{x^2}{a^2}-1.$$

$$\frac{y}{b}=\pm\sqrt{\frac{x^2}{a^2}-1},$$

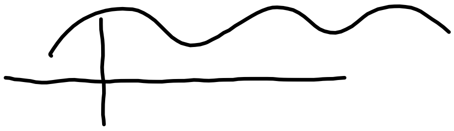
$$y=\pm b\cdot\sqrt{\frac{x^2}{a^2}-1}$$

$$y=\pm\frac{b}{a}\cdot x\cdot\sqrt{1-\underbrace{\frac{a^2}{x^2}}_{\to 0 \text{ når } x^2\to\infty}}$$

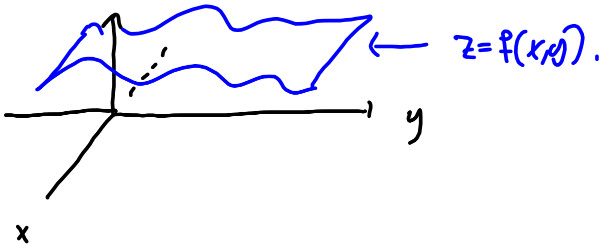
3.7. Grafisk fremstilling av skalarfelter.

Skalarfelt: $f : \mathbb{R}^n \longrightarrow \mathbb{R}$.

For $n=1$:

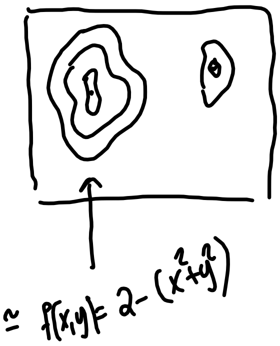


For $n=2$:

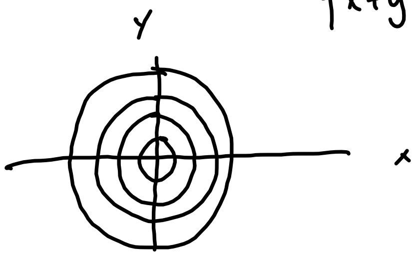


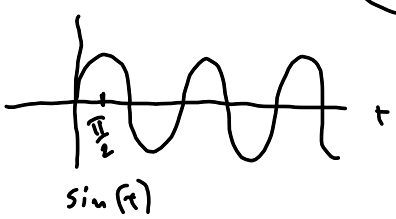
EKS: $f(x,y) = \sin(x^2+y^2)$.

Nivåkurver: En nivåkurve for f er en mengde $\{(x,y) : f(x,y) = c\}$.

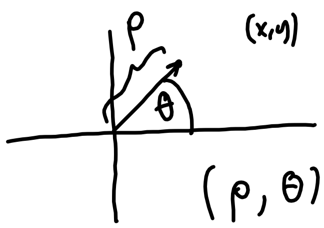


I eksempelet: nivåkurvene er $\{x^2+y^2 = c\}$.





Polarkoordinater:



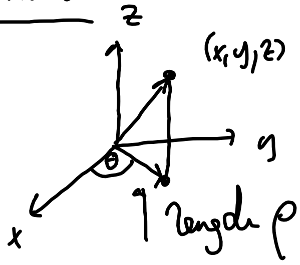
$x = \rho\cos\theta$, $y = \rho\sin\theta$.

$$f(\rho,\theta) = \sin\left(\rho^2\cos^2\theta + \rho^2\sin^2\theta\right) = \sin(\rho^2).$$

Koordinater i tre variabler

Sylinderkoordinater:

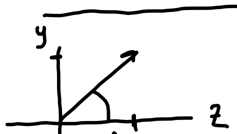
Polarkoordinater for x og y, og behold z.



lengde ρ av proj. på (x,y)-planet.

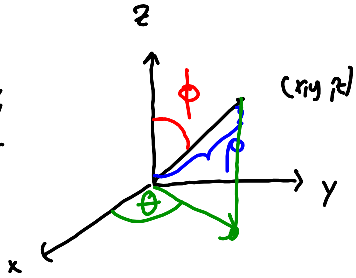
(ρ, θ, z),

Kulekoordinater:



x: lengde $\cdot \cos\theta$

y: " $\sin\theta$



ϕ er vinkelen mod positiv z-akse.

ρ er lengden.

θ er vinkelen mellom projeksjonen på (x,y)-planet og den positive x-aksen.

Finner x, y og z gitt (ρ, θ, ϕ).

$$\boxed{z = \rho \cdot \cos\phi.}$$

Har
$$x^2 + y^2 + z^2 = \rho^2$$
$$x^2 + y^2 = \rho^2 - z^2 = \rho^2 - \rho^2\cos^2\phi$$
$$= \rho^2(1 - \cos^2\phi) = \rho^2\sin^2\phi$$
$$\sqrt{x^2 + y^2} = \rho \cdot \sin\phi$$

$$\boxed{x = \rho \cdot \sin\phi \cdot \cos\theta}$$

$$\boxed{y = \rho\sin\phi \cdot \sin\theta.}$$

```
>> x=-2:0.05:2;
>> y=x;
>> [x,y]=meshgrid(x,y);
>> mesh(x,y,sin(x.^2+y.^2))
>> x=-5:0.05:5;
>> y=x;
>> [x,y]=meshgrid(x,y);
>> mesh(x,y,sin(x.^2+y.^2))
>>
>>
>> %Eksempel Kulekoordinater
>>
>> x=linspace(0,pi,100);
>> y=linspace(0,2*pi,100);
>> [x,y]=meshgrid(x,y);
>> surf(sin(x).*cos(y),sin(x).*sin(y),cos(x))
>>
>> % Her er x phi og y er theta.
>>
```