Midtveiseksamen 2013

Oppgave 1: $F(x,y) = (x^2 y, xy^4)$

$(1,1)$.

Linearisering: $F(1,1) + F'(1,1) \cdot \begin{pmatrix} x-1 \\ y-1 \end{pmatrix}$

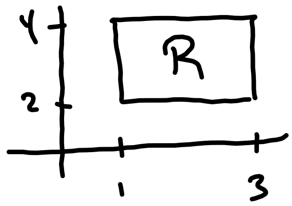
- $F(1,1) = (1,1)$.
- $F'(x,y) = \begin{bmatrix} 2xy & x^2 \\ y^4 & 4xy^3 \end{bmatrix}$

$$F'(1,1) = \begin{bmatrix} 2 & 1 \\ 1 & 4 \end{bmatrix}$$

Linearisering: $(1,1) + \begin{bmatrix} 2 & 1 \\ 1 & 4 \end{bmatrix} \begin{pmatrix} x-1 \\ y-1 \end{pmatrix}$

$$= (1,1) + \left(2(x-1) + (y-1), x-1 + 4(y-1)\right)$$
$$= (1,1) + (2x + y - 3, x + 4y - 5)$$
$$= (-2,-4) + (2x+y, x+4y).$$

Oppgave 2:



$$F(x,y) = (1,3) + A(x,y),$$

$$A = \begin{bmatrix} 2 & 7 \\ 3 & 1 \end{bmatrix}$$

Arealet av $F(R)$?

Arealet blir skalert med $|\det(A)|$,

så arealet blir: $4 \cdot |-19| = \underline{\underline{76}}$

Oppgave 3 $\quad x^2 - 10x + y^2 - 6y + 30 = 0$

$$(x-5)^2 - 25 + (y-3)^2 - 9 + 30 = 0$$

$$(x-5)^2 + (y-3)^2 = 4.$$

Sirkel med senter $(5,3)$ og radius 2.

Oppgave 4: $\quad L: \mathbb{R}^2 \to \mathbb{R}^2$ s.a.

$$L(5 \cdot \vec{e}_1) = (2,4)$$

$$L(\vec{e}_2) = (-1,3)$$

Finn matrisen til L.

$$A = \begin{bmatrix} 2/5 & -1 \\ 4/5 & 3 \end{bmatrix}$$

$T(\vec{e}_1) = \begin{pmatrix} a \\ b \end{pmatrix}$

$T(\vec{e}_2) = \begin{pmatrix} c \\ d \end{pmatrix}$

$B = \begin{bmatrix} a & c \\ b & d \end{bmatrix}$

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Oppgave 5 $\vec{r}(t) = (2t^2, \sin t)$.

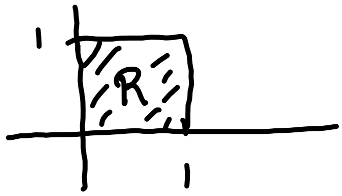
$\vec{a}(t)$?

$\vec{a}(t) = \vec{r}\,''(t)$.

$\vec{r}\,'(t) = (4t, \cos t)$

$\vec{r}\,''(t) = (4, -\sin t)$.

Oppgave 6



$\mathbb{R}^2$

$f(x,y) = x^3y + 5xy^2$

$$\iint_R f(x,y)\,dx\,dy = \int_0^1\int_0^1 x^3y + 5xy^2\,dx\,dy$$

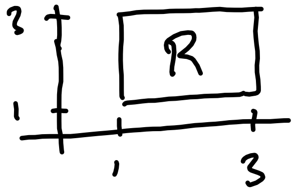
$$= \int_0^1 \left(\int_0^1 x^3y + 5xy^2\,dx \right) dy$$

$$= \int_0^1 \left[\frac{1}{4}x^4y + \frac{5}{2}x^2y^2 \right]_0^1 dy$$

$$= \int_0^1 \frac{1}{4}y + \frac{5}{2}y^2\,dy$$

$$= \frac{1}{8} + \frac{5}{6} = \underline{\underline{\frac{23}{24}}}$$

Oppgave 7 :



$$f(x,y) = 2x + 5y$$

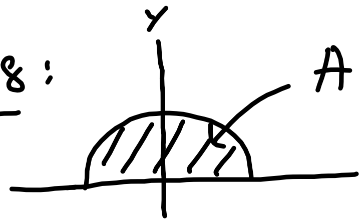
Finn arealet av grafen til f over R.

$$\text{Arealet} = \iint_R \sqrt{1 + \frac{\partial f}{\partial x}(x,y)^2 + \frac{\partial f}{\partial y}(x,y)^2}\, dx\,dy$$

$$= \int_1^3 \int_1^3 \sqrt{1+4+25}\, dx\, dy$$

$$= \underline{\underline{\sqrt{30} \cdot 4}} \,.$$

Oppgave 8 :



$$\iint_A x^2 y \, dx\, dy = \int_0^1 \int_0^\pi r^2 \cos^2 t \cdot r \cdot \sin t \overset{r}{\downarrow} \, dt\, dr$$

$$\left(\text{polar: } x = r\cdot\cos t,\ y = r\sin t \quad r \in [0,1],\ t \in [0,\pi] \right) = \int_0^1 \int_0^\pi r^4 \cdot \cos^2 t \cdot \sin t \, dt\, dr$$

$$= \frac{1}{5} \int_0^\pi \cos^2 t \cdot \sin t \, dt \,.$$

$$(\cos^3 t)' = 3\cos^2 t \cdot (-\sin t)$$

$$= \frac{1}{5} \cdot \left[-\frac{1}{3}\cos^3 t \right]_0^\pi$$

$$= \frac{1}{15}(1+1) = \underline{\underline{\frac{2}{15}}}$$

Oppgave 9: $F: \mathbb{R}^2 \to \mathbb{R}^2$

$F(0,0) = (0,0)$.

$F'(0,0) = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$.

$g: \mathbb{R}^2 \to \mathbb{R}$

$g'(0,0) = (2,3)$.

$h(x,y) = g(F(x,y))$

$h'(0,0)$?

Kjerneregel: $h'(0,0) = g'(F(0,0)) \cdot F'(0,0)$

$= g'(0,0) \cdot F'(0,0)$

$= (2,3) \cdot \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \underline{\underline{(11, 16)}}$

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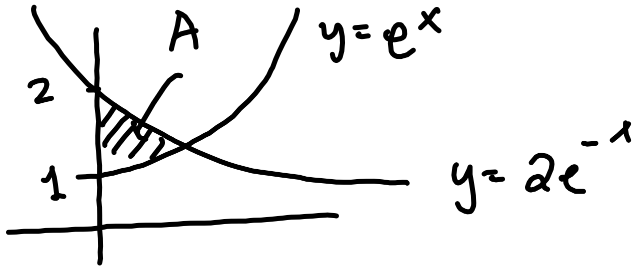
Oppgave 10 :



$$\iint_A y \, dx\, dy \; ?$$

$$= \int_0^2 \left(\int_{e^x}^{2e^{-x}} y \, dy \right) dx$$

$$\left(\text{Integrasjonsgrense:} \quad \begin{aligned} e^x &= 2e^{-x} \\ e^{2x} &= 2 \\ 2x &= \ln 2 \\ x &= \tfrac{1}{2} \ln 2 . \end{aligned} \right)$$

$$= \frac{1}{2} \int_0^{\frac{1}{2}\ln 2} \left[y^2 \right]_{e^x}^{2e^{-x}} dx = \frac{1}{2} \int_0^{\frac{1}{2}\ln 2} 4e^{-2x} - e^{2x} \, dx$$

$$= \frac{1}{2} \left[-2e^{-2x} - \frac{1}{2} e^{2x} \right]_0^{\frac{1}{2}\ln 2}$$

$$= \frac{1}{2} \left[-2e^{-\ln 2} - \frac{1}{2} e^{\ln 2} + 2 + \frac{1}{2} \right]$$

$$= \frac{1}{2} \cdot \left[\cancel{-1} \cancel{-1} + \cancel{2} + \frac{1}{2} \right] = \underline{\underline{\frac{1}{4}}} .$$

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oppgave 11) $f(x,y) = x^2y + 5xy^2$

S er grafen til f i $\mathbb{R}^2$.

Finn tangentplanet til S i $(1,1,f(1,1))$:

$$\left[\begin{array}{l} \frac{\partial f}{\partial x}(x,y) = 2xy + 5y^2 \\ \frac{\partial f}{\partial y}(x,y) = x^2 + 10xy \end{array}\right]$$

Tangentplant: $z = f(1,1) + \frac{\partial f}{\partial x}(1,1)\cdot(x-1) + \frac{\partial f}{\partial y}(1,1)\cdot(y-1)$

$= 6 + 7(x-1) + 11\cdot(y-1)$

$= 6-7-11 + 7x + 11y$

$= \underline{\underline{-12 + 7x + 11y}}.$

oppgave 12) $\vec{r}(t) = (t^2, t^3)$, $t \in [0, 2]$

Finn lengden til kurven.

$$l = \int_0^2 v(t)\,dt = \int_0^2 |\vec{r}\,'(t)|\,dt$$

$$\left(r'(t) = (2t, 3t^2), \quad |\vec{r}\,'(t)| = \sqrt{4t^2 + 9t^4} \right)$$

$$= \int_0^2 \sqrt{4t^2 + 9t^4}\,dt = \int_0^2 t \cdot \sqrt{4 + 9t^2}\,dt$$

substitusjon

$$= \left[\frac{2}{3} \cdot (4 + 9t^2)^{3/2} \cdot \frac{1}{18} \right]_0^2$$

$$= \frac{1}{27}\left(40^{3/2} - 4^{3/2}\right)$$

oppgave 13 $\vec{r}(t) = (\cos t, 3\sin t)$, $t \in [0, \frac{\pi}{2}]$.

$f(x,y) = x \cdot y$

$\int_C f \cdot ds$?

$\boxed{\vec{r}'(t) = (-\sin t, 3\cos t)}$

$$= \int_0^{\pi/2} 3 \cdot \cos t \cdot \sin t \cdot |\vec{r}'(t)| \, dt$$

$$= 3\int_0^{\pi/2} (\sin^2 t + 9\cos^2 t)^{1/2} \cdot \cos t \cdot \sin t \, dt .$$

$$= 3\int_0^{\pi/2} \left(\sin^2 t + 9(1-\sin^2 t)\right)^{1/2} \cos t \cdot \sin t$$

$$= 3\int_0^{\pi/2} (9 - 8\sin^2 t)^{1/2} \cos t \cdot \sin t \, dt .$$

substitusjon

$$= 3 \cdot \left[(9 - 8\sin^2 t)^{3/2} \cdot \frac{2}{3} \cdot \left(-\frac{1}{16}\right) \right]_0^{\pi/2}$$

$= \ldots\ldots$

oppgave 14) C e samme kurve som i 13)

$\vec{r}(t) = (\cos(t), 3\sin(t))$, $t \in [0, \frac{\pi}{2}]$

$\phi(x,y) = x^2 + \cos(xy)$, $\vec{F} = \nabla\phi$

$\int_C \vec{F} \cdot d\vec{r}$?

$$\int_C \vec{F} \cdot d\vec{r} = \int_C \nabla\phi \cdot d\vec{r} = \phi(\vec{r}(\pi/2)) - \phi(\vec{r}(0))$$

$\Big(\vec{r}(\frac{\pi}{2}) = (0, 3)$
$\vec{r}(0) = (1, 0) \Big)$

$= 1 - 2 = \underline{\underline{-1}}.$