1) $\vec{r}(t) = (-\sin(2t), t, \cos(2t))$

$0 \leq t \leq \pi$

$$\sqrt{x'(t)^2 + y'(t)^2 + z'(t)}$$

$$= \sqrt{(-2\cos(2t))^2 + 1^2 + (-2\sin(2t))^2}$$

$$= \sqrt{4\cos^2(2t) + 1 + 4\sin^2(2t)}$$

$$= \sqrt{4+1} = \sqrt{5}!$$

$$L(0,\pi) = \int_0^\pi \sqrt{(x'(t))^2 + (y'(t))^2 + (z'(t))^2}\, dt$$

$$= \int_0^\pi \sqrt{5}\, dt = \underline{\underline{\sqrt{5}\,\pi}}$$

2) $\vec{F}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x^2+y^2 \\ x^2-y^2 \end{pmatrix}$

$\vec{F}\begin{pmatrix} 1 \\ -1 \end{pmatrix} = \begin{pmatrix} 2 \\ 0 \end{pmatrix}$

$\vec{F}'\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2x & 2y \\ 2x & -2y \end{pmatrix}$ $\quad \vec{F}'\begin{pmatrix} 1 \\ -1 \end{pmatrix} = \begin{pmatrix} 2 & -2 \\ 2 & 2 \end{pmatrix}$

$$T_a \vec{F}(\vec{x}) = \vec{F}(\vec{a}) + \vec{F}'(\vec{a})(\vec{x} - \vec{a})$$

her: $\begin{pmatrix} 2 \\ 0 \end{pmatrix} + \begin{pmatrix} 2 & -2 \\ 2 & 2 \end{pmatrix}\left(\begin{pmatrix} x \\ y \end{pmatrix} - \begin{pmatrix} 1 \\ -1 \end{pmatrix}\right)$

$$= \begin{pmatrix} 2 \\ 0 \end{pmatrix} + \begin{pmatrix} 2 & -2 \\ 2 & 2 \end{pmatrix} \begin{pmatrix} x-1 \\ y+1 \end{pmatrix}$$

3) $\vec{v}_1 = \begin{pmatrix} 2 \\ 3 \\ 7 \end{pmatrix}$ $\quad \vec{v}_2 = \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix}$

$$\begin{pmatrix} 2 & 1 & 2 \\ 3 & 0 & -2 \\ 7 & 2 & 2 \end{pmatrix} \sim \begin{pmatrix} 1 & \frac{1}{2} & 1 \\ 3 & 0 & -2 \\ 7 & 2 & 2 \end{pmatrix} \sim \begin{pmatrix} 1 & \frac{1}{2} & 1 \\ 0 & -\frac{3}{2} & -5 \\ 0 & -\frac{3}{2} & -5 \end{pmatrix}$$

$$\sim \begin{pmatrix} 1 & \frac{1}{2} & 1 \\ 0 & 1 & \frac{10}{3} \\ 0 & 0 & 0 \end{pmatrix} \sim \begin{pmatrix} 1 & 0 & -\frac{2}{3} \\ 0 & 1 & \frac{10}{3} \\ 0 & 0 & 0 \end{pmatrix}$$

Derfor: $\begin{pmatrix} 2 \\ -2 \\ 2 \end{pmatrix} = -\frac{2}{3}\vec{v}_1 + \frac{10}{3}\vec{v}_2$

4) $A = \begin{pmatrix} 1 & 0 & 4 \\ 2 & 2 & 3 \\ 1 & 2 & -1 \end{pmatrix} \sim \begin{pmatrix} 1 & 0 & 4 \\ 0 & 2 & -5 \\ 0 & 2 & -5 \end{pmatrix} \sim \begin{pmatrix} 1 & 0 & 4 \\ 0 & 1 & -\frac{5}{2} \\ 0 & 0 & 0 \end{pmatrix}$

Søyle 1 og 2 er pivotsøyler, men ikke søyle 3.

Determinanten til A er 0.

5) Egenverdiene er 2 og 3, siden matrisen er triangulær, og da står egenverdiene på diagonalen.

6) $\vec{F}(x,y,z) = (\underbrace{2xyz^3}_{\frac{\partial\phi}{\partial x}}, \underbrace{x^2z^3}_{\frac{\partial\phi}{\partial y}}, \underbrace{3x^2yz^2}_{\frac{\partial\phi}{\partial z}})$

$$\phi(x,y,z) = x^2yz^3 + C(y,z)$$
$$\phi(x,y,z) = x^2yz^3 + D(x,z)$$
$$\phi(x,y,z) = x^2yz^3 + E(x,y)$$

Vi kan sette $C = D = E = 0$, slik at

$\phi(x,y,z) = x^2yz^3$.

$\vec{F}$ blir da konservativt.

7) $\vec{r}(t) = (t, \frac{2\sqrt{2}}{3}t^{3/2}, \frac{1}{2}t^2)$ $\quad 0 \le t \le 1$.

$$\sqrt{x'(t)^2 + y'(t)^2 + z'(t)^2} = \sqrt{1 + (\sqrt{2}t^{\frac{1}{2}})^2 + t^2}$$
$$= \sqrt{1 + 2t + t^2} = t+1$$

$$\int_C x^2\,ds = \int_0^1 x(t)^2\sqrt{x'(t)^2 + y'(t)^2 + z'(t)^2}\,dt$$

$$= \int_0^1 t^2(t+1)\,dt = \int_0^1 (t^3+t^2)\,dt$$

$$= \left[\frac{1}{4}t^4 + \frac{1}{3}t^3\right]_0^1 = \frac{1}{4} + \frac{1}{3} = \underline{\underline{\frac{7}{12}}}$$

8) $\vec{F}(x,y,z) = (x^2, y^2, z^2)$ $\quad \vec{r}(t) = (\cos t, \sin t, t)$

$0 \leq t \leq 2\pi$

metode 1: $\vec{F}(\vec{r}(t)) = (\cos^2 t, \sin^2 t, t^2)$

$\vec{r}\,'(t) = (-\sin t, \cos t, 1)$

$\vec{F}(\vec{r}(t)) \cdot \vec{r}\,'(t) = -\cos^2 t \sin t + \sin^2 t \cos t + t^2$

$$\int_C \vec{F} \cdot d\vec{r} = \int_0^{2\pi} \vec{F}(\vec{r}(t)) \cdot \vec{r}\,'(t)\,dt$$

$$= \int_0^{2\pi} \Big(\underbrace{-\cos^2 t \sin t}_{\substack{u=\cos t \\ du=-\sin t\,dt}} + \underbrace{\sin^2 t \cos t}_{\substack{u=\sin t \\ du=\cos t\,dt}} + t^2\Big)\,dt$$

$$= \left[\frac{1}{3}\cos^3 t + \frac{1}{3}\sin^3 t + \frac{1}{3}t^3\right]_0^{2\pi} = \underline{\underline{\frac{8}{3}\pi^3}}$$

metode 2: $\vec{F}$ er konservativt med potensialfunksjon

$$\phi(x,y,z) = \frac{1}{3}x^3 + \frac{1}{3}y^3 + \frac{1}{3}z^3$$

Da får vi: $\int_C \vec{F} \cdot d\vec{r} = \phi(\vec{r}(b)) - \phi(\vec{r}(a))$

$$= \phi(\vec{r}(2\pi)) - \phi(\vec{r}(0))$$

$$= \phi(1,0,2\pi) - \phi(1,0,0)$$

$$= \frac{1}{3} + 0 + \frac{8\pi^3}{3} - \frac{1}{3} - 0 - 0$$

$$= \frac{8}{3}\pi^3.$$

9) $A = \begin{pmatrix} 5 & -1 \\ -1 & 5 \end{pmatrix}$

$$\det(\lambda I - A) = (\lambda-5)(\lambda-5) - 1 = \lambda^2 - 10\lambda + 24 = 0$$

$$\lambda = \frac{10 \pm \sqrt{100-96}}{2} = \frac{10 \pm 2}{2} = 5 \pm 1$$

$$\lambda_1 = 4, \quad \lambda_2 = 6.$$

egenvektor for $\lambda_1 = 4$:

$$4I - A = \begin{pmatrix} -1 & 1 \\ 1 & -1 \end{pmatrix} \sim \begin{pmatrix} 1 & -1 \\ 0 & 0 \end{pmatrix}$$

en egenvektor må oppfylle $x_1 - x_2 = 0 \Leftrightarrow x_1 = x_2$

egenvektor med $x_1 = 1$: $\vec{v}_1 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

egenvektor for $\lambda_2 = 6$

$$6I - A = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \sim \begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix}$$

en egenvektor må oppfylle $x_1 + x_2 = 0 \Leftrightarrow x_2 = -1$

egenvektor med $x_1 = -1$: $\vec{v}_2 = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$

10) utvidet matrise:

$$\begin{pmatrix} 2 & 2 & 1 \\ 1 & 3 & 3 \\ 6 & 10 & a \end{pmatrix} \sim \begin{pmatrix} 1 & 3 & 3 \\ 2 & 2 & 1 \\ 6 & 10 & a \end{pmatrix}$$

$$\sim \begin{pmatrix} 1 & 3 & 3 \\ 0 & -4 & -5 \\ 0 & -8 & a-18 \end{pmatrix} \sim \begin{pmatrix} 1 & 3 & 3 \\ 0 & 1 & \frac{5}{4} \\ 0 & -8 & a-18 \end{pmatrix}$$

$$\sim \begin{pmatrix} 1 & 3 & 3 \\ 0 & 1 & \frac{5}{4} \\ 0 & 0 & a-8 \end{pmatrix}$$

siste søyle pivotsøyle $\Leftrightarrow$ ingen løsning $\Leftrightarrow$ $a \neq 8$

$a = 8$: entydig løsning, siden de to første søylene er pivotsøyler.

11) $9x^2 - 36x + 16y^2 + 64y = 44$

$$9(x^2 - 4x + 4) + 16(y^2 + 4y + 4) = 44 + 36 + 64 = 144 = 12^2$$

$$9(x-2)^2 + 16(y+2)^2 = 12^2$$

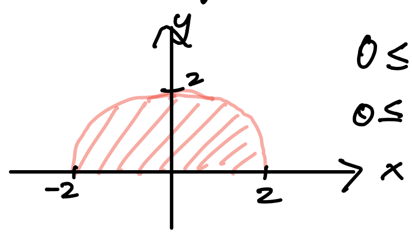
$$\frac{(x-2)^2}{4^2} + \frac{(y+2)^2}{3^2} = 1$$

ellipse med sentrum $(2,-2)$
halvakser 4 og 3.

12)

$$\iint_A x^2 y^2 \, dx\, dy$$

$x^2+y^2 \leq 4 \quad y \geq 0$



$0 \leq r \leq 2$
$0 \leq \theta \leq \pi$

$$= \int_0^{\pi} \left[\int_0^2 r^2\cos^2\theta \, r^2\sin^2\theta \, r \, dr \right] d\theta$$

$$= \int_0^{\pi} \left[\int_0^2 r^5 \cos^2\theta \sin^2\theta \, dr \right] d\theta$$

$$= \int_0^{\pi} \left[\frac{1}{6} r^6 \cos^2\theta \sin^2\theta \right]_0^2 d\theta$$

$$= \int_0^{\pi} \frac{32}{3} \cos^2\theta \sin^2\theta \, d\theta$$

$\sin 2\theta = 2\sin\theta\cos\theta$
$\frac{1}{4}\sin^2 2\theta = \sin^2\theta\cos^2\theta$

$$= \int_0^{\pi} \frac{8}{3} \sin^2 2\theta \, d\theta =$$

$\cos 2\theta = 1 - 2\sin^2\theta$
$\sin^2\theta = \frac{1}{2}(1-\cos 2\theta)$

$$= \int_0^{\pi} \frac{8}{3} \cdot \frac{1}{2}(1-\cos 4\theta)\, d\theta$$

$$= \int_0^{\pi} \frac{4}{3}(1-\cos 4\theta)\, d\theta = \left[\frac{4}{3}\theta - \frac{1}{3}\sin 4\theta \right]_0^{\pi}$$

$$= \underline{\frac{4\pi}{3}}$$

13)

$$\iiint_A xyz \, dx\,dy\,z = \int_0^1 \left[\int_0^2 \left[\int_0^3 xyz \, dz \right] dy \right] dx$$

$$= \int_0^1 \left[\int_0^2 \left[\tfrac{1}{2} xyz^2 \right]_0^3 dy \right] dx$$

$$= \int_0^1 \left[\int_0^2 \tfrac{9}{2} xy \, dy \right] dx$$

$$= \int_0^1 \left[\tfrac{9}{4} xy^2 \right]_0^2 dx$$

$$= \int_0^1 9x \, dx = \left[\tfrac{9}{2} x^2 \right]_0^1 = \underline{\underline{\tfrac{9}{2}}}$$

14)

$$\iint_A \frac{1}{(x^2+y^2)^3} \, dx\,dy \qquad A = \{(x,y) \in \mathbb{R}^2 : x^2+y^2 \geq 1\}$$

polarkoordinater: $r \geq 1$.

$$\| $$

$$\lim_{n\to\infty} \int_0^{2\pi} \left[\int_1^n \frac{1}{(r^2)^3} r \, dr \right] d\theta$$

$$= \lim_{n\to\infty} \int_0^{2\pi} \left[\int_1^n r^{-5} dr \right] d\theta$$

$$= \lim_{n\to\infty} \int_0^{2\pi} \left[\frac{1}{-4} r^{-4} \right]_1^n d\theta = \lim_{n\to\infty} \int_0^{2\pi} \tfrac{1}{4}(1 - n^{-4}) \, d\theta$$

$$= \lim_{n\to\infty} \tfrac{2\pi}{4}(1-n^{-4}) = \frac{2\pi}{4} = \underline{\underline{\frac{\pi}{2}}}$$

15) $\vec{F}(x,y) = P(x,y)\vec{i} + Q(x,y)\vec{j}$

$$= \left(y^2 + e^{\cos x},\ x^2 - \tan y\right)$$

C enhetssirkelen orientert mot klokka.

$$\int_C P\,dx + Q\,dy \overset{\text{Greens}}{=} \iint_D \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right) dx\,dy$$

$$= \iint_D (2x - 2y)\,dx\,dy$$

$$= \int_0^{2\pi} \left[\int_0^1 (2r\cos\theta - 2r\sin\theta)\, r\,dr\right] d\theta$$

$$= \int_0^{2\pi} \left[\int_0^1 2r^2(\cos\theta - \sin\theta)\,dr\right] d\theta$$

$$= \int_0^{2\pi} \left[\frac{2}{3} r^3 (\cos\theta - \sin\theta)\right]_0^1 d\theta$$

$$= \int_0^{2\pi} \frac{2}{3}(\cos\theta - \sin\theta)\,d\theta$$

$$= \left[\frac{2}{3}(\sin\theta + \cos\theta)\right]_0^{2\pi} = \underline{\underline{0}}$$