13 Trippelintegralet $\int\int\int_A xyz\,dxdydz$ over rektanglet $R = [0,1] \times [0,2] \times [0,3]$ er

Velg ett alternativ

- $7/2$
- $3/2$
- 2
- $9/2$ ✔
- 1

$R = [0,1] \times [0,2] \times [0,3]$

$$\iiint_R xyz \, dx\,dy\,dz = \int_0^1 \int_0^2 \int_0^3 xyz \, dz\,dy\,dx$$

$$= \int_0^1 \int_0^2 xy \left. \frac{z^2}{2} \right|_0^3 dy\,dx$$

$$= \int_0^1 \int_0^2 \frac{9}{2} xy \, dy\,dx$$

$$= \frac{9}{2} \int_0^1 \int_0^2 xy \, dy\,dx$$

$$= \frac{9}{2} \int_0^1 x \left. \frac{y^2}{2} \right|_0^2 dx$$

$$= \frac{9}{2} \int_0^1 x \frac{4}{2} \, dx$$

$$= 9 \int_0^1 x \, dx$$

$$= 9 \cdot \frac{1}{2} = \frac{9}{2}$$

D

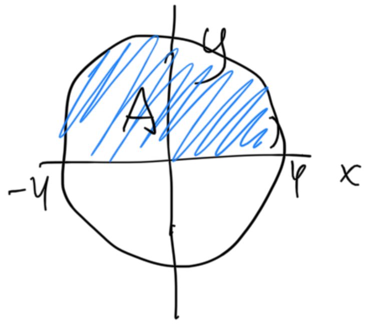
12 Dobbeltintegralet $\int\int_A x^2y^2\,dxdy$ over området A beskrevet ved $x^2+y^2\leq 4$ og $y\geq 0$ er

Velg ett alternativ

- $\pi/2$
- 0
- $4\pi/3$ ✔
- π
- 2π

Hint: Her kan du få bruk for at $\sin(2x) = 2\sin x\cos x$, samt en av de andre trigonometriske identitetene i formelsamlingen.

$$x^2 + y^2 \le 4$$
$$y \geq 0$$



I polarkoordinater: $0 \le r \le 2$

$0 \le \theta \le \pi$

$$\iint_A x^2 y^2 \, dx\,dy = \int_0^2 \int_0^\pi r^2 \cos^2\theta \cdot r^2 \sin^2\theta \cdot r \, dr \, d\theta$$

$$= \int_0^2 \int_0^\pi r^5 \cos^2\theta \cdot \sin^2\theta \, dr \, d\theta$$

$$= \int_0^\pi \frac{r^6}{6} \cos^2\theta \sin^2\theta \Big|_0^2 \, d\theta$$

$$= \frac{64}{6} \int_0^\pi \cos^2\theta \sin^2\theta \, d\theta$$

$\cos^2\theta = 1 - \sin^2\theta$

$$= \frac{64}{6} \int_0^\pi \sin^2\theta - \sin^4\theta \, d\theta$$

$$= \frac{64}{6} \cdot \frac{1}{32} \Big[4\theta - \sin(4\theta) \Big]_0^\pi$$

$$= \frac{4\pi}{3}$$

$\rightsquigarrow$ C

14 Det uegentlige integralet $\int\int_A \frac{1}{(x^2+y^2)^3} dxdy$, der $A = \{(x,y) \in \mathbb{R}^2 : x^2+y^2 \geq 1\}$,

Velg ett alternativ

- divergerer
- er lik $\pi/2$ ✔
- er lik 0
- er lik $2\pi/5$
- er lik π

Bruker polarkoordinater:

A = området der $r \geq 1$

$$\rightsquigarrow \iint_A \frac{1}{(x^2+y^2)^3} dxdy = \int_1^\infty \int_0^{2\pi} \frac{1}{(r^2)^3} \cdot r \, dr \, d\theta$$

$$= 2\pi \lim_{R\to\infty} \int_1^R r^{-5} \, dr$$

$$= 2\pi \lim_{R\to\infty} -\frac{1}{4} r^{-4} \Big|_1^R$$

$$= 2\pi \lim_{R\to\infty} \left(0 - \left(-\frac{1}{4} 1\right)\right)$$

$$= 2\pi \cdot \frac{1}{4} = \pi/2 \rightsquigarrow B$$

Oppgave 13. (4 poeng) La $f(x,y) = x^2 + y^2$ og la S være grafen til f i $\mathbb{R}^3$. Tangentplanet til S i punktet $(1, 2, f(1,2))$ er definert ved

A) $z = 4x + 2y - 5$

B) $z = 4x - 2y + 5$

C) $z = 2x + 4y - 5$

D) $z = 2x + 2y + 5$

E) $z = x - y + 1$

$f(x,y) = x^2 + y^2 \qquad a = (1, 2, 5)$

Tangentplanet til S

$\nabla f = (2x, 2y)$

$\nabla f(a) = (2, 4)$

$$\rightsquigarrow T_a f = f(a) + \nabla f(a) \cdot \begin{pmatrix} x-1 \\ y-2 \end{pmatrix}$$

$$= 5 + \begin{pmatrix} 2 \\ 4 \end{pmatrix} \cdot \begin{pmatrix} x-1 \\ y-2 \end{pmatrix}$$

$$= 5 + 2(x-1) + 4(y-2)$$

$$= 2x + 4y - 5$$

$\rightsquigarrow$ C

Oppgave 14. (4 poeng)La S være samme flate som i forrige oppgave. Da er normalvektoren til S i punktet $(1, 2, f(1, 2))$ gitt ved

A) (-4,2,1)

B) (-2,-4,1)

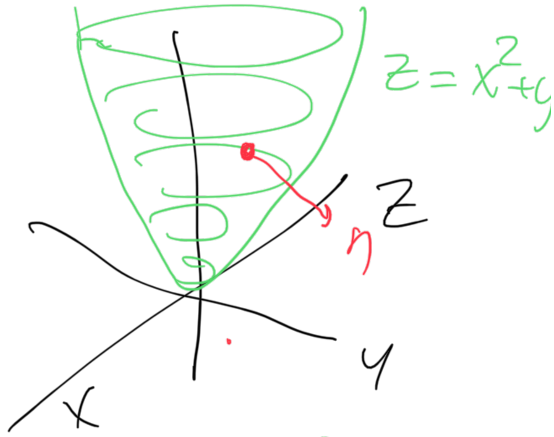
C) (-2,-4,-1)

D) (-2,-2,1)

E) (-1,1,1)

Normal vektoren n:

$$n = \begin{pmatrix} \underbrace{\begin{matrix}\frac{\partial f}{\partial x}\\[4pt] \frac{\partial f}{\partial y}\end{matrix}}_{\nabla f} \\ -1\end{pmatrix} = \left.\begin{pmatrix}2x\\2y\\-1\end{pmatrix}\right|_{\substack{x=1\\ y=2}} = \begin{pmatrix}2\\4\\-1\end{pmatrix}$$



C

Oppgave 15. (4 poeng) La S være samme flate som i de to forrige oppgavene. Da er arealet av den delen av S som ligger over området $x^2 + y^2 \leq 1$ lik

A) $5\pi/6$

B) $(\pi/6)(5\sqrt{5})$

C) $(\pi/6)(2\sqrt{5} - 1)$

D) $(\pi/6)(\sqrt{5} - 1)$

E) $(\pi/6)(5\sqrt{5} - 1)$

$$\text{Area} = \iint_S 1 \cdot dS$$

$$z = f(x,y) = x^2 + y^2$$

$$dS = \sqrt{1 + \left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2}\, dx\,dy$$

$$= \sqrt{1 + 4x^2 + 4y^2}\, dx\,dy$$

$$A = \{x^2 + y^2 \le 1\}$$

$$\rightsquigarrow \text{area} = \iint_A \sqrt{1+4x^2+4y^2}\, dx\,dy$$

$u = 1 + 4r^2$

$du = 8r\,d$

$$= \int_0^{2\pi} \int_0^1 \sqrt{1+4r^2} \cdot r\, dr$$

$$= \frac{2\pi}{8} \left[\frac{2}{3}(1+4r)^{3/2}\right]_0^1$$

$$= \frac{\pi}{4} \cdot \frac{2}{3}\left(5^{3/2} - 1\right)$$

$$= \frac{\pi}{6}\left(5^{3/2} - 1\right)$$

ms E