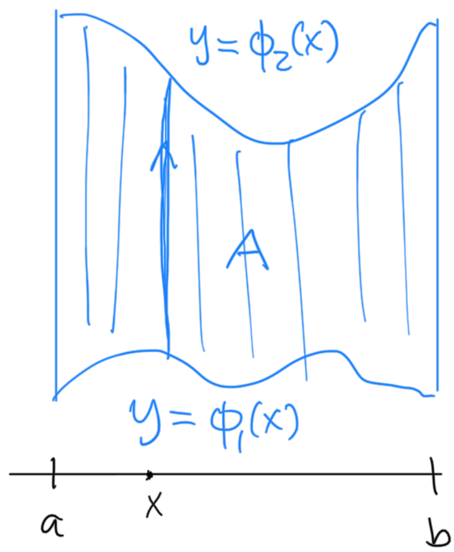
Multippel integrasjon

Dobbeltintegraler

$$R = [a,b] \times [c,d] \quad \rightsquigarrow \quad \iint_R f(x,y)\,dx\,dy = \int_a^b \left(\int_c^d f(x,y)\,dy \right) dx$$

$A \subset \mathbb{R}^2$ type I:

$$\iint_A f\,dx\,dy = \int_a^b \int_{\phi_1(x)}^{\phi_2(x)} f\,dy\,dx$$



$A \subset \mathbb{R}^2$ type II

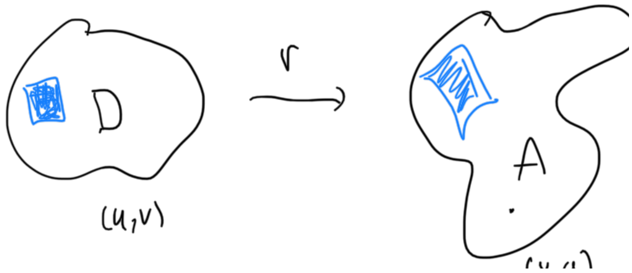
$$\iint_A f\,dx\,dy = \int_c^d \int_{\phi_1(y)}^{\phi_2(y)} f\,dx\,dy$$

generelt; del opp områder i type I og II

Skifte av variabel

jacobifaktoren

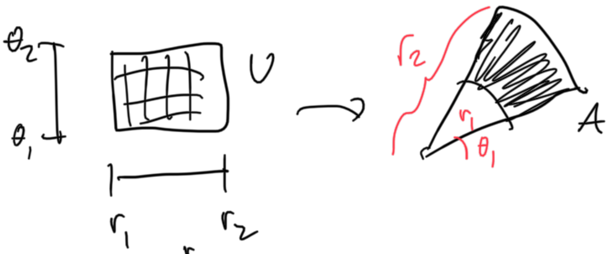
$$\iint_A f(x,y)\,dx\,dy = \iint_D f(r(u,v)) \cdot |\det r'(u,v)|\,du\,dv$$



koordinater

polarkoordinater (x,y)

$$\boxed{\begin{aligned} x &= r\cos\theta \\ y &= r\sin\theta \end{aligned}}$$



$$\iint_A f\,dx\,dy = \int_{r_1}^{r_2}\int_{\theta_1}^{\theta_2} f(r\cos\theta, r\sin\theta)\,\underline{r}\,d\theta\,dr$$

Trippelintegraler

$$\iiint_R f(x,y,z)\,dx\,dy\,dz = \int_{a_1}^{a_2}\int_{b_1}^{b_2}\int_{c_1}^{c_2} f(x,y,z)\,dz\,dy\,dx$$

$$R = [a_1,a_2]\times[b_1,b_2]\times[c_1,c_2]$$

Skifte av variabel:

$$\iiint_A f(x,y,z)\,dx\,dy\,dz = \iiint_D f(T(u,v,w))\left|\frac{\partial(x,y,z)}{\partial(u,v,w)}\right| du$$

$x = x(u,v,w)$
$y = y(u,v,w)$
$z = z(u,v,w)$

$$\left|\begin{matrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} & \frac{\partial x}{\partial w} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} & \frac{\partial y}{\partial w} \\ \frac{\partial z}{\partial u} & \frac{\partial z}{\partial v} & \frac{\partial z}{\partial w} \end{matrix}\right|$$

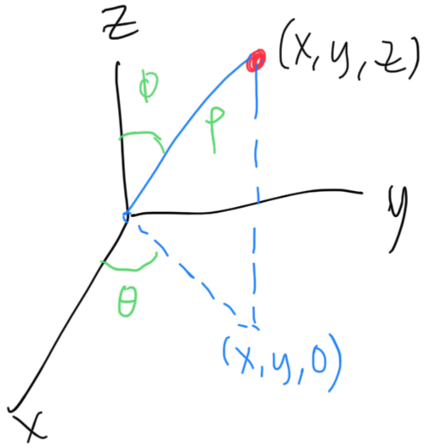
Sylinderkoordinater:

$x = r\cos\theta$
$y = r\sin\theta$
$z = z$

$$\rightsquigarrow \left|\frac{\partial(x,y,z)}{\partial(r,\theta,z)}\right| = r$$

Kulekoordinater

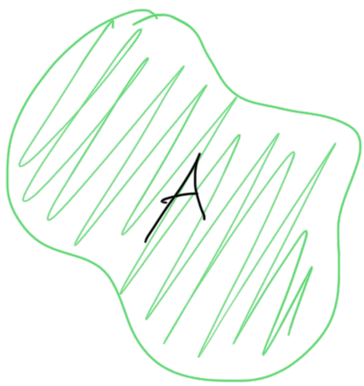
$$x = \rho \sin\phi \cos\theta$$
$$y = \rho \sin\phi \sin\theta$$
$$z = \rho \cos\phi$$



$$\rightsquigarrow \left|\frac{\partial(x,y,z)}{\partial(\rho,\phi,\theta)}\right| = \rho^2 \sin\phi$$

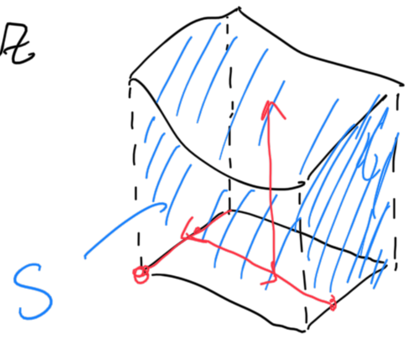
Areal

$$\text{areal}(A) = \iint_A 1\, dx\, dy$$

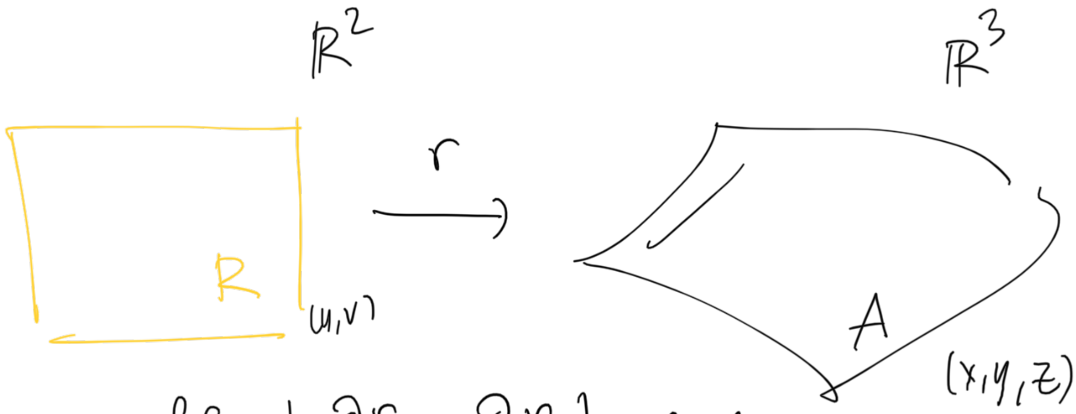


Volum

$$\text{volum}(S) = \iiint_S 1\, dx\, dy\, dz$$



Overflateareal



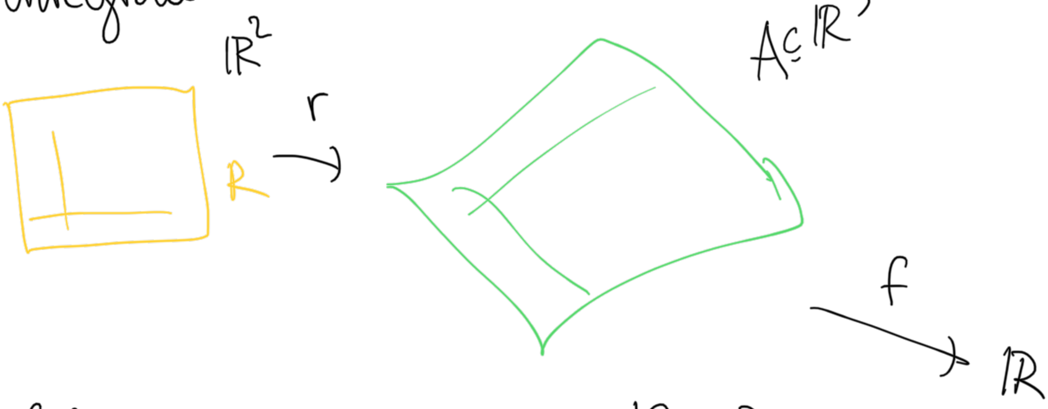
$$\text{areal}(A) = \iint_R \left| \frac{\partial r}{\partial u} \times \frac{\partial r}{\partial v} \right| du\,dv$$

$r(u,v) = (x(u,v), y(u,v), z(u,v))$

For flater på formen $z = f(x,y)$:

$$\text{areal}(A) = \iint_R \sqrt{1 + \left(\frac{\partial f}{\partial u}\right)^2 + \left(\frac{\partial f}{\partial v}\right)^2}\, du\,dv$$

Flateintegraler



$$\rightsquigarrow \iint f\, dS := \iint_R f(r(u,v)) \cdot \left| \frac{\partial r}{\partial u} \times \frac{\partial r}{\partial v} \right| du\,dv$$