

Anvendelser av diagonalisering

Demografi

La $x = \left(\begin{array}{c} \text{andel nordmenn som bor i by} \\ \text{-----} \text{ " -----} \\ \text{på landet} \end{array} \right)$, f.eks. $x = \begin{pmatrix} 1/2 \\ 1/2 \end{pmatrix}$.

x er en sannsynlighetsfordeling: $x \in \mathbb{R}^n$, $x_k \geq 0 \forall k$, $\sum_{k=1}^n x_k = 1$.

Fra ett år til neste vil f.eks. $x \rightsquigarrow \begin{pmatrix} a & b \\ c & d \end{pmatrix} x$

f.eks. $a = 0.98$, $c = 0.02$,
 $d = 0.96$, $b = 0.04$

$$x_{r+1} = A x_r, \quad A = \begin{pmatrix} 0.98 & 0.04 \\ 0.02 & 0.96 \end{pmatrix}$$

$$\Rightarrow x_r = A^r x_0$$

$$\boxed{x_r = A^r x_0}, \quad A = R D R^{-1}. \quad \text{Da er}$$

$$x_r = R D^r \underbrace{R^{-1} x_0}_{=y} = R D^r y$$

$$= \lambda_1^r \alpha_1 v_1 + \lambda_2^r \alpha_2 v_2$$

Om $|\lambda_1| > |\lambda_2|$, vil $x_r \approx \lambda_1^r \alpha_1 v_1$ for $r \gg 1$

Vi for $\lambda_1 = 1$, $\lambda_2 < 1$, nå $x_r \approx \alpha_1 v_1$ for $r \gg 1$

$$y = \begin{pmatrix} \alpha_1 \\ \alpha_2 \end{pmatrix}$$

$$R = (v_1 \quad v_2)$$

$$D = \begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix}$$

Kredittvurdering

- *Kredittvurderingsbyråer:* Standard & Poor's, Moody, Finch, ...
- Vurderer en låntagers evne/sannsynlighet for å betale tilbake et lån.
- Låntager rangeres på en skala (A/B/C/D)
- Byråene estimerer hvert år sannsynligheten for å endre kredittvurdering.

Table 1: One-year transition probabilities matrix.

Initial ratings	Ratings at year-end							
	AAA	AA	A	BBB	BB	B	CCC	Default
AAA	0.9366	0.0583	0.0040	0.0009	0.0002	0	0	0
AA	0.0066	0.9172	0.0694	0.0049	0.0006	0.0009	0.0002	0.0002
A	0.0007	0.0225	0.9176	0.0518	0.0049	0.0020	0.0001	0.0004
BBB	0.0003	0.0026	0.0483	0.8924	0.0444	0.0081	0.0016	0.0023
BB	0.0003	0.0006	0.0044	0.0666	0.8323	0.0746	0.0105	0.0107
B	0	0.0010	0.0032	0.0046	0.0572	0.8362	0.0384	0.0594
CCC	0.0015	0	0.0029	0.0088	0.0191	0.1028	0.6123	0.2526
Default	0	0	0	0	0	0	0	1.0000

Source: Standard & Poor's, January 2001

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