Jonas Irgens Kylling

jonasik@math.uio.no

Zariski's cancellation problem

Let X be an algebraic variety (the zeros of a set of polynomial equations) over $\mathbb{C}$ and $\mathbb{A}^n_{\mathbb{C}}$ affine n-space. If $X \times \mathbb{A}^1_{\mathbb{C}} = \mathbb{A}^{n+1}_{\mathbb{C}}$, is then $X = \mathbb{A}^n_{\mathbb{C}}$? This is known as a Zariski's cancellation problem. If X is a point, a curve, or a surface (i.e., the dimension of X is 0, 1 or 2) the answer is yes. However, for higher dimensions the answer is unknown and is expected to be no. In fact, a particular family of surfaces known as Koras–Russel threefolds are expected to be counterexamples. If we instead of the complex numbers work over a finite field there are counterexamples constructed by Gupta.

This project could involve some of the following:

- Formulate Zariski's cancellation problem [3].
- Study Koras–Russel threefolds and explain why they are candidate counterexamples.
- Explain why Zariski cancellation holds in low dimension [1], [2].
- Study the counterexample of Gupta over finite fields [4].
- Search for an isomorphism $X \times \mathbb{A}^1 \cong \mathbb{A}^4$, where X is a Koras–Russel threefold, for instance with the help of a computer (i.e., an isomorphism of polynomial rings with 5 (resp. 4) generators).

References

[1] Shreeram S. Abhyankar, William Heinzer, and Paul Eakin. "On the uniqueness of the coefficient ring in a polynomial ring". In: *J. Algebra* 23 (1972), pp. 310–342. ISSN: 0021-8693. URL: `https://doi.org/10.1016/0021-8693(72)90134-2`.

[2] Anthony J. Crachiola and Leonid G. Makar-Limanov. “An algebraic proof of a cancellation theorem for surfaces”. In: *J. Algebra* 320.8 (2008), pp. 3113–3119. ISSN: 0021-8693. URL: `https://doi.org/10.1016/j.jalgebra.2008.03.037`.

[3] Neena Gupta. “A survey on Zariski cancellation problem”. In: *Indian J. Pure Appl. Math.* 46.6 (2015), pp. 865–877. ISSN: 0019-5588. URL: `https://doi.org/10.1007/s13226-015-0154-3`.

[4] Neena Gupta. “On the cancellation problem for the affine space $\mathbb{A}^3$ in characteristic p”. In: *Invent. Math.* 195.1 (2014), pp. 279–288. ISSN: 0020-9910. URL: `https://doi.org/10.1007/s00222-013-0455-2`.