

THE 36 QUANTUM OFFICERS OF EULER

As legend has it, russian empress Catherine the Great asked a mathematician at her court whether it is possible to arrange 36 officers of her army each having one out of six ranks and belonging to one out of six regiments in a 6×6 square such that no two officers with the same rank or belonging to the same regiment stand in the the same row or column. That mathematician was Leonard Euler and after studying the problem, he came to the conclusion that it was impossible. In more mathematical terms, he conjectured that there is no pair of orthogonal Latin squares of order 6, and over 100 years after Euler's death this conjecture was finally proven.

Another 100 years later, mathematicians and physicistis studying quantum entanglement came up with a quantum variant of the problem: Here, a pair of orthogonal quantum Latin squares corresponds to a particular unitary operator acting on a tensor product of finite vector spaces and it can informally be thought of as Latin squares where entangled quantum states are written in the entries and orthogonality relations replace the usual requirements on the entries of Latin squares. Somewhat surprisingly, last year it has been shown that there exist quantum orthogonal Latin squares of order 6 (see article by Rather et al. below).

Besides this being a nice puzzle, there are many applications of quantum orthogonal Latin squares in the theory of entanglement and to design quantum error correcting codes. The goal of the project is to understand these fascinating results and to learn about their connections to other parts of quantum information science.

Prerequisites: The only prerequisite is a solid understanding of linear algebra. Prior knowledge in quantum physics, tensor products or combinatorics is *not* needed.

Literature:

- Starting point is the article “Thirty-six entangled officers of Euler: Quantum solution to a classically impossible problem” by Rather et al. (see <https://arxiv.org/pdf/2104.05122.pdf>).
- To get a rough idea, you can also check out the popular article ”Euler’s 243-Year-Old ‘Impossible’ Puzzle Gets a Quantum Solution” on Quanta Magazine.