

POSITIVE MAPS ON LOW-DIMENSIONAL MATRIX ALGEBRAS

Consider linear maps $\phi : \mathcal{M}_n \rightarrow \mathcal{M}_m$ mapping the algebra of $n \times n$ matrices with complex entries $\mathcal{M}_n$ into the algebra of $m \times m$ matrices with complex entries $\mathcal{M}_m$. An elementary problem is to determine when such a map is *positive*, i.e., when does it send positive semidefinite $n \times n$ matrices to positive semidefinite $m \times m$ matrices. Despite the fact that this problem is easy to state, it has turned out quite difficult for general values of n and m .

For certain small dimensions the positive linear maps between matrix algebras do have a nice characterization. This is the case when $n = m = 2$ or when $n = 2$ and $m = 3$ (and $n = 3$ and $m = 2$). These results have important consequences for the theory of quantum entanglement, where they allow for an easy criterion in low dimensions for when bipartite quantum states are entangled. The goal of this project is to understand the characterizations of positive maps in these low-dimensional cases (which is surprisingly involved) and possibly to find better proofs for these results. If time permits, we can go further to explore some open problems in the theory of entanglement and positive maps, which recently came up in the context of quantum information processing.

Prerequisites: The only prerequisite is a solid understanding of linear algebra. Prior knowledge in quantum physics, tensor products or combinatorics is *not* needed.

Literature:

- Starting point is the article “Positive maps of low dimensional matrix algebras” by S.L. Woronowicz (see <https://www.sciencedirect.com/science/article/abs/pii/003448776900380#!>).
- Another good reference is the book “Positive Linear Maps of Operator Algebras” by E. Størmer.