

The Jones polynomial for knots

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Goal

- Low-level: understand different ways to define the Jones polynomial for knots
- High-level: introduction to ideas behind the quantum topology

Description

The *Jones polynomial* is an invariant of *knots*, i.e., different ways to embed the circle in a 3-dimensional space without self-intersection, and up to continuous deformation. This is a key notion in the field of low-dimensional topology, and it is desirable to find good invariants for knots that are easy to compute while strong enough to distinguish as many different ones as possible.

About 40 years ago, Jones found way to define Laurent polynomial $V_K(t)$ for knots K through the study of inclusions of infinite-dimensional algebras (subfactor theory). Despite the complicated background, $V_K(t)$ itself has a hands-on algorithm to compute based on a knot diagrams of K , and turned out to be a very powerful invariant of knots. This attracted active research from diverse viewpoints, and soon people found intriguing connections with statistical mechanics, quantum field theory, and beyond, which led to different ways to define $V_K(t)$ via Kauffman bracket (due to Kauffman), representation of quantum groups (due to Reshetikhin and Turaev), and noncommutative gauge theory (due to Witten).

Our goal is to learn the accessible parts of the theory starting from a survey by Jones [Jon05]. We aim to reach a bit deeper with extra materials, such as the structure of the *Temperley–Lieb algebras* from [Jon83], or the *Kauffman polynomials* from [Kau90].

References

- [Jon83] V. F. R. Jones, *Index for subfactors*, Invent. Math. **72** (1983), no. 1, 1–25.
- [Jon05] V. F. R. Jones, *The Jones Polynomial*, available at the author’s webpage.
- [Kau90] Louis H. Kauffman, *An invariant of regular isotopy*, Trans. Amer. Math. Soc. **318** (1990), no. 2, 417–471.