

MAT2000 PROJECT: THE GRIGORCHUK GROUP

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In this project we will become familiar with important techniques and notions in geometric group theory. One such notion is the *growth* of a group. The *growth function* γ_S associated with a group G with finite generating set S can be defined as follows: $\gamma_S(n)$ is the number of elements $g \in G$ for which we may write $g = s_1 s_2 \cdots s_k$, where $s_i \in S$ and $k \leq n$. It is then important to understand how this function behaves asymptotically as $n \rightarrow \infty$. If γ_S grows like a polynomial function, we say G has *polynomial growth*. It is also not very difficult to find examples of groups for which the growth function grows like an exponential function, in which case we say the group has *exponential growth*. This is the case for the free group on 2 generators, for example.

In 1968, Milnor posed the following problem: Do there exist groups which have intermediate growth, that is, for which the growth function grows faster than any polynomial, yet sub-exponentially? This was answered in the positive by Grigorchuk in 1984, through the construction of what is now known as the (first) Grigorchuk group.

The goal of this project would be to study the Grigorchuk group and prove that it has intermediate growth.

Another direction would be to study Gromov's theorem on polynomial growth, which says that the class of finitely generated groups with polynomial growth coincides with the class of virtually nilpotent groups.

Excellent sources for this material are

- “Amenability of discrete groups by examples” by Juschenko
- “Geometric Group Theory” by Drutu and Kapovich

both of which are available for free online.

Prerequisites: You should have taken a course in abstract algebra covering basic group theory, for example MAT2200 Groups, rings and fields