

MAT2000: Kuratowski's sets in topology

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A *topological space* consists of a set of points X together with a *topology*, i.e. a collection $\mathcal{T}$ of subsets of X called the *open subsets* of X obeying that:

1. The empty set $\emptyset$ and X belong to $\mathcal{T}$.
2. If $A, B \in \mathcal{T}$ then $A \cap B \in \mathcal{T}$.
3. Given a collection $\{A_i\}_{i \in I} \subset \mathcal{T}$ then $\bigcup_{i \in I} A_i \in \mathcal{T}$.

A topology on a set allows for describing several notions familiar to the real line $\mathbb{R}$ such as closure, limits, continuity or connectedness. The purpose of the project is to get familiarized with these topological notions and with the ideas relevant to understand the statement of the Kuratowski closure-complement theorem:

Theorem 1. *Let $(X, \mathcal{T})$ be a topological space. Given a subset $A \subset X$, there are at most 14 sets obtained from A by considering closures and complements.*

In a second stage of the project, one should look into multiple examples. In particular, an example where all 14 sets are different. One could further study the Kuratowski monoid of a space and related classification results.

References

- [GJ08] B.J. Gardner and M. Jackson, *The Kuratowski closure-complement theorem*, New Zealand Journal of Mathematics, Volume 38, 9–44, (2008).
- [Links:] https://en.wikipedia.org/wiki/Kuratowski%27s_closure-complement_problem