

MAT2000 Project:

Sampling and interpolation in Paley–Wiener spaces

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Say you have a reasonably nice function defined on $\mathbb{R}$ that you want to represent discretely by sampling it along a sequence of equally spaced points. How close must these points be in order for your function to be reconstructible from the samples? This basic question is answered by the celebrated sampling theorem of Nyquist–Shannon–Whittaker: The sampling rate must be more than twice the size of the “highest frequency” in your function.

In this project, we aim to obtain an understanding of some of the mathematical foundations of signal processing. Mathematically, the sampling theorem can be viewed as a statement about Paley–Wiener spaces, which are function spaces central to harmonic analysis. The Paley–Wiener space $PW_K(\mathbb{R}^d)$ consists of complex-valued functions on $\mathbb{R}$ whose Fourier transform vanishes outside the compact set $K \subseteq \mathbb{R}^d$. In addition to proving the sampling theorem, another goal will be to look at more general sets of sampling and interpolation in Paley–Wiener spaces and their relation to Beurling’s notion of density.

Suggested reading: <https://links.uwaterloo.ca/amath731docs/sampling.pdf>, *Sampling Theory in Fourier and Signal Analysis* by J. R. Higgins.

Prerequisites: Preferably both MAT2400 (Real analysis) and MAT2410 (Introduction to Complex Analysis). I also recommend taking MAT3400 (Linear analysis with applications) parallel to writing the project.

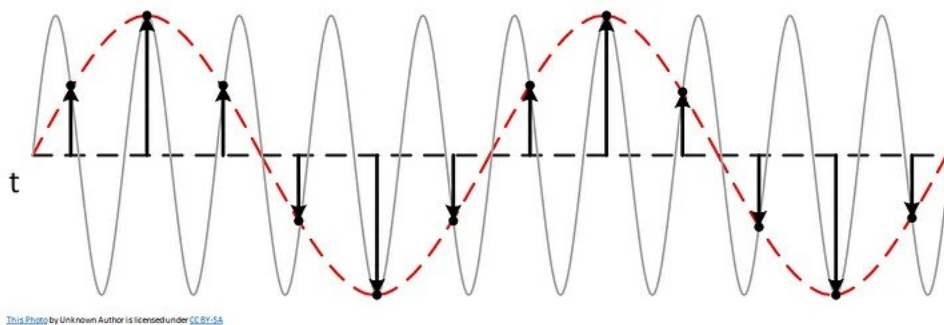


Figure 1: The grey function agrees with the red dashed function on the given sampling set. A higher sample rate is needed to determine the grey signal.