

FOUNDATIONS OF ANALYSIS AND CONSTRUCTIVE ANALYSIS

1. PROJECT OUTLINE

The theoretical/methodological basis for main stream mathematics, and in particular analysis, is the theory of sets. In proving various results we argue via contradictions, we assume that mathematical statements are either true or false, and we frequently prove existence results without being able to get our hands on the objects whose existence are claimed (see e.g. [2]). On the other hand, Errett Bishop has developed mathematical analysis from a constructive point of view. In *Foundations of constructive analysis* [1] he develops mathematical analysis from his constructive point of view; all mathematical objects whose existence we claim must be constructed. As an example, the familiar Rolle's Theorem: *For a differentiable function f on the interval $[a, b]$ satisfying $f(a) = f(b)$ there exists $c \in (a, b)$ with $f'(c) = 0$* , is no longer true; presumably there is no universal method for producing such a point c (however there is a constructive counterpart of this result). This result is classically based on the *least upper bound property* of the real numbers, which does not hold constructively; there exist sets whose elements are either 0 or 1, that do not have least upper bounds. To quote Bishop:

We are not interested in properties of the positive integers that have no descriptive meaning for finite man. When a man proves a positive integer to exist, he should show how to find it. If God has mathematics of his own that needs to be done, let him do it himself.

In spite of this quote this project is not meant as constructive propaganda. Rather, the goal is to study and compare the bases of classical and constructive analysis, in order to get a clearer understanding of the assumptions behind our activities.

REFERENCES

1. Bishop, Errett; *Foundations of constructive analysis*. McGraw-Hill Book Co., New York-Toronto-London, 1967, xiii+370 pp.
2. Tao, Terrence; *Analysis I*. Texts Read. Math., **37** Hindustan Book Agency, New Delhi, 2022, xvi+355 pp.