

MAT2000 Project: Quantum Channels and Markov Kernels

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Description

In classical probability theory, Markov transition kernels are used to model the stochastic evolution of probability measures and play a central role in the theory of Markov processes. In quantum theory, the evolution of states is described by quantum channels, which are completely positive, trace-preserving maps acting on density operators. The aim of this project is to study quantum channels as noncommutative analogues of classical Markov kernels.

The project will include a review of the basic objects of quantum probability, including density operators, normal states, and completely positive maps. Fundamental structural results such as Kraus representations and Stinespring dilations are studied. Particular attention is given to the parallels between quantum channels and reduced dynamics of larger deterministic systems, and their interpretation in terms of noise and loss of information.

Goal

- Understand Markov transition kernels and their formulation in terms of Markov operators.
- Understand quantum channels as completely positive, trace-preserving maps and their structural properties.
- Clarify the relationship between classical Markov kernels and quantum channels.

References

- S. Attal, *Lecture notes*, (2022). Chapter 6 on Quantum Channels is the most important but chapters 1,2,4,5 are also relevant.
- O. Kallenberg, *Foundations of Modern Probability*, 2nd edition, Springer, 2002. Chapter 3 and 11 in particular.