UNIVERSITY OF OSLO

Faculty of mathematics and natural sciences

Examination in MAT 2200 — Groups, rings and fields

Day of examination: Friday 6. june 2008.

Examination hours: 9.00 – 12.00.

This problem set consists of 2 pages.

Appendices: None

Permitted aids: None

Please make sure that your copy of the problem set is complete before you attempt to answer anything.

Problem 1

a

Find three different abelian groups of order 8 and give reasons why all abelian groups of order 8 are isomorphic to one of these.

b

Find a non-abelian group of order 8. Give reasons, why it is not abelian

c

How many subgroups of order 8 are there in S_4, the symmetric group on 4 elements. How many of these are abelian? Give reasons for your answer.

Problem 2

a

Show that $x^5 + 4x^3 + 2$ is irreducible over $\mathbb{Q}$.

b

Show that x^2+1, x^2+x-1 and x^2-x-1 are the only irreducible polynomials of degree 2 over $\mathbb{Z}_3$, and show that $x^5 + 4x^3 + 2$ is irredusible over $\mathbb{Z}_3$.

c

Explain why $F = \mathbb{Z}_3[x]/(x^5 + 4x^3 + 2)$ form a field that contains $\mathbb{Z}_3$. How many elements are there in F?

(Continued on page 2.)

Problem 3

Let $f(x) = x^4 - 2x^2 - 3$.

a

Find the splitting field E of $f(x)$ over $\mathbb{Q}$. What is $[E : \mathbb{Q}]$ and $G(E/\mathbb{Q})$?

b

Find an element $a \in E$ such that $E = \mathbb{Q}(a)$ (simple extension). Find the minimal polynomial ($Irr(a, \mathbb{Q})$) of a over $\mathbb{Q}$.