to show a function is one to one is precisely to show that it does not carry two points into just one point. Thus, proving a function is one to one becomes more natural in the two-to-two terminology.

SECTION 1

1. $-i$ **3.** $-i$ **5.** $23+7i$
7. $17-15i$ **9.** $-4+4i$
11. $2\sqrt{13}$ **13.** $\sqrt{2}\left(-\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}}i\right)$
15. $\sqrt{34}\left(\frac{-3}{\sqrt{34}}+\frac{5}{\sqrt{34}}i\right)$
17. $\frac{1}{\sqrt{2}}\pm\frac{1}{\sqrt{2}}i, -\frac{1}{\sqrt{2}}\pm\frac{1}{\sqrt{2}}i$ **19.** $3i, \pm\frac{3\sqrt{3}}{2}-\frac{3}{2}i$
21. $\sqrt{3}\pm i, \pm 2i, -\sqrt{3}\pm i$ **23.** 4 **25.** $\frac{3}{8}$ **27.** $\sqrt{2}$
29. 11 **31.** 5 **33.** 1, 7
35. $\zeta^0 \leftrightarrow 0, \zeta^3 \leftrightarrow 7, \zeta^4 \leftrightarrow 4, \zeta^5 \leftrightarrow 1, \zeta^6 \leftrightarrow 6, \zeta^7 \leftrightarrow 3$
37. With $\zeta \leftrightarrow 4$, we must have $\zeta^2 \leftrightarrow 2$, $\zeta^3 \leftrightarrow 0$, and $\zeta^4 \leftrightarrow 4$ again, which is impossible for a one-to-one correspondence.
39. Multiplying, we obtain

$$z_1z_2 = |z_1||z_2|[(\cos\theta_1\cos\theta_2 - \sin\theta_1\sin\theta_2) + (\cos\theta_1\sin\theta_2 + \sin\theta_1\cos\theta_2)i]$$

and the desired result follows at once from Exercise 38 and the equation $|z_1||z_2| = |z_1z_2|$.

SECTION 2

1. e, b, a **3.** a, c. $*$ is not associative.
5. Top row: d; second row: a; fourth row: c, b.
7. Not commutative, not associative
9. Commutative, associative
11. Not commutative, not associative
13. 8, 729, $n^{[n(n+1)/2]}$
17. No. Condition 2 is violated. **19.** Yes
21. No. Condition 1 is violated.
23. **a.** Yes. **b.** Yes
25. Let $S = \{?, \Delta\}$. Define $*$ and $*'$ on S by $a * b = ?$ and $a *' b = \Delta$ for all $a, b \in S$. (Other answers are possible.)
27. True **29.** True
31. False. Let $f(x) = x^2$, $g(x) = x$, and $h(x) = 2x + 1$. Then
$(f(x) - g(x)) - h(x) = x^2 - 3x - 1$ but
$f(x) - (g(x) - h(x)) = x^2 - (-x - 1) = x^2 + x + 1$.
33. True **35.** False. Let $*$ be $+$ and let $*'$ be on $\mathbb{Z}$.

SECTION 3

1. **i.** ϕ must be one to one. **ii.** $\phi[S]$ must be all of S'.
iii. $\phi(a * b) = \phi(a) *' \phi(b)$ for all $a, b \in S$.
3. No, because ϕ does not map $\mathbb{Z}$ onto $\mathbb{Z}'$. $\phi(n) \neq 1$ for all $n \in \mathbb{Z}$.
5. Yes. **7.** Yes **9.** Yes
11. No, because $\phi(x^2) = \phi(x^2 + 1)$.
13. No, because $\phi(f) = x + 1$ has no solution $f \in F$.