15. No, because $\phi(f) = 1$ has no solution $f \in F$.

17. **a.** $m * n = mn - m - n + 2$; identity element 2

b. $m * n = mn + m + n$; identity element 0

19. **a.** $a * b = \frac{1}{3}(ab + a + b - 2)$; identity element 2

b. $a * b = 3ab - a - b + \frac{2}{3}$; identity element $\frac{2}{3}$

25. No. If $\langle S, * \rangle$ has a left identity element e_L and a right identity element e_R, then $e_L = e_R$. (It is our practice to omit proofs from answers.)

SECTION 4

1. No. $\mathscr{G}_3$ fails. **3.** No. $\mathscr{G}_1$ fails. **5.** No. $\mathscr{G}_1$ fails.

7. The group $\langle U_{1000}, \cdot \rangle$ of solutions of $z^{1000} = 1$ in $\mathbb{C}$ under multiplication has 1000 elements.

9. An equation of the form $x * x * x * x = e$ has four solutions in $\langle U, \cdot \rangle$, one solution in $\langle \mathbb{R}, + \rangle$, and two solutions in $\langle \mathbb{R}^*, \cdot \rangle$.

11. Yes **13.** Yes

15. No. The matrix with all entries 0 is upper triangular, but has no inverse.

17. Yes.

19. (Proofs are omitted.) **c.** $-1/3$

21. 2, 3. (It gets harder for 4 elements, where the answer is *not* 4.)

25. **a.** F **c.** T **e.** F **g.** T **i.** F

SECTION 5

1. Yes **3.** Yes **5.** Yes **7.** $\mathbb{Q}^+$ and $\{\pi^n \mid n \in \mathbb{Z}\}$ **9.** Yes

11. No. Not closed under multiplication.

13. Yes

15. **a.** Yes **b.** No. It is not even a subset of $\tilde{F}$.

17. **a.** No. Not closed under addition. **b.** Yes

19. **a.** Yes **b.** No. The zero constant function is not in $\tilde{F}$.

21. **a.** $-50, -25, 0, 25, 50$ **b.** $4, 2, 1, 1/2, 1/4$ **c.** $1, \pi, \pi^2, 1/\pi, 1/\pi^2$

23. All matrices $\begin{bmatrix} 1 & n \\ 0 & 1 \end{bmatrix}$ for $n \in \mathbb{Z}$

25. All matrices of the form $\begin{bmatrix} 4^n & 0 \\ 0 & 4^n \end{bmatrix}$ or $\begin{bmatrix} 0 & -2^{2n+1} \\ -2^{2n+1} & 0 \end{bmatrix}$ for $n \in \mathbb{Z}$

27. 4 **29.** 3 **31.** 4 **33.** 2 **35.** 3

39. **a.** T **c.** T **e.** F **g.** F **i.** T

SECTION 6

1. $q = 4, r = 6$ **3.** $q = -7, r = 6$ **5.** 8 **7.** 60

9. 4 **11.** 16 **13.** 2 **15.** 2 **17.** 6 **19.** 4

21. An infinite cyclic group

23.