SECTION 11

1.

Element	*Order*	*Element*	*Order*
(0, 0)	1	(0, 2)	2
(1, 0)	2	(1, 2)	2
(0, 1)	4	(0, 3)	4
(1, 1)	4	(1, 3)	4

The group is not cyclic

3. 2 **5.** 9 **7.** 60

9. {(0, 0), (0, 1)}, {(0, 0), (1, 0)}, {(0, 0), (1, 1)}

11. {(0, 0), (0, 1), (0, 2), (0, 3)}
{(0, 0), (0, 2), (1, 0), (1, 2)}
{(0, 0), (1, 1), (0, 2), (1, 3)}

13. $\mathbb{Z}_{20} \times \mathbb{Z}_3, \mathbb{Z}_{15} \times \mathbb{Z}_4, \mathbb{Z}_{12} \times \mathbb{Z}_5, \mathbb{Z}_5 \times \mathbb{Z}_3 \times \mathbb{Z}_4$

15. 12

17. 120

19. 180

21. $\mathbb{Z}_8, \mathbb{Z}_2 \times \mathbb{Z}_4, \mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathbb{Z}_2$

23. $\mathbb{Z}_{32}, \mathbb{Z}_2 \times \mathbb{Z}_{16}, \mathbb{Z}_4 \times \mathbb{Z}_8, \mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathbb{Z}_8, \mathbb{Z}_2 \times \mathbb{Z}_4 \times \mathbb{Z}_4,$
$\mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathbb{Z}_4, \mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathbb{Z}_2$

25. $\mathbb{Z}_9 \times \mathbb{Z}_{121}, \mathbb{Z}_3 \times \mathbb{Z}_3 \times \mathbb{Z}_{121}, \mathbb{Z}_9 \times \mathbb{Z}_{11} \times \mathbb{Z}_{11}, \mathbb{Z}_3 \times \mathbb{Z}_3 \times \mathbb{Z}_{11} \times \mathbb{Z}_{11}$

29. a.

n	2	3	4	5	6	7	8
number of groups	2	3	5	7	11	15	22

b. i) 225 **ii)** 225 **iii)** 110

31. a. It is abelian when the arrows on both n-gons have the same (clockwise or counterclockwise) direction.
b. $\mathbb{Z}_2 \times \mathbb{Z}_n$
c. When n is odd.
d. The dihedral group D_n.

33. $\mathbb{Z}_2$ is an example.

35. S_3 is an example.

37. The numbers are the same. **41.** $\{-1, 1\}$

SECTION 12

1. a. The only isometries of $\mathbb{R}$ leaving a number c fixed are the reflection through c that carries $c + x$ to $c - x$ for all $x \in \mathbb{R}$, and the identity map.
b. The isometries of $\mathbb{R}^2$ that leave a point P fixed are the rotations about P through any angle θ where $0 \le \theta < 360^\circ$ and the reflections across any axis that passes through P.
c. The only isometries of $\mathbb{R}$ that carry a line segment into itself are the reflection through the midpoint of the line segment (see the answer to part (a)) and the identity map.
d. The isometries of $\mathbb{R}^2$ that carry a line segment into itself are a rotation of 180° about the midpoint of the line segment, a reflection in the axis containing the line segment, a reflection in the axis perpendicular to the line segment at its midpoint, and the identity map.
e. The isometries of $\mathbb{R}^3$ that carry a line segment into itself include rotations through any angle about an axis that contains the line segment, reflections across any plane that contains the line segment, and reflection across the plane perpendicular to the line segment at its midpoint.