35. Let $\phi(m, n) = (m, 0)$ for $(m, n) \in \mathbb{Z}_2 \times \mathbb{Z}_4$.
37. Let $\phi(n) = \rho_n$ for $n \in \mathbb{Z}_3$, using our notation in the text for elements of S_3.
39. Let $\phi(m, n) = 2m$.
41. Viewing D_4 as a group of permutations, let $\phi(\sigma) = (1, 2)$ for odd $\sigma \in D_4$ and $\phi(\sigma)$ be the identity for even $\sigma \in D_4$.
43. Let $\phi(\sigma) = (1, 2)$ for odd $\sigma \in S_4$ and $\phi(\sigma)$ be the identity element for even $\sigma \in S_4$.
51. The image of ϕ is $\langle a \rangle$, and Ker(ϕ) must be some subgroup $n\mathbb{Z}$ of $\mathbb{Z}$.
53. $hk = kh$ **55.** h^n must be the identity e of G.

SECTION 14

1. 3 **3.** 4 **5.** 2 **7.** 2
9. 4 **11.** 3 **13.** 4 **15.** 1
21. **a.** When working with a factor group G/H, you would let a and b be elements of G, not elements of G/H. The student probably does not understand what elements of G/H look like and can write nothing sensible concerning them.
b. We must show that G/H is abelian. Let aH and bH be two elements of G/H.
23. **a.** T **c.** T **e.** T **g.** T **i.** T
29. $\{\rho_0, \mu_1\}$, $\{\rho_0, \mu_2\}$, and $\{\rho_0, \mu_3\}$
35. Example: Let $G = N = S_3$, and let $H = \{\rho_0, \mu_1\}$. Then N is normal in G, but $H \cap N = H$ is not normal in G.

SECTION 15

1. $\mathbb{Z}_2$ **3.** $\mathbb{Z}_4$ **5.** $\mathbb{Z}_4 \times \mathbb{Z}_8$ **7.** $\mathbb{Z}$ **9.** $\mathbb{Z}_3 \times \mathbb{Z} \times \mathbb{Z}_4$
11. $\mathbb{Z}_2 \times \mathbb{Z}$ **13.** $\mathbb{Z}(D_4) = C = \{\rho_0, \rho_2\}$
15. $Z(S_3 \times D_4) = \{(\rho_0, \rho_0), (\rho_0, \rho_2)\}$, using the notations for these groups in Section 8, $C = A_3 \times \{\rho_0, \rho_2\}$.
19. **a.** T **c.** F **e.** F **g.** F **i.** T
21. $\{f \in F^* \mid f(0) = 1\}$
23. Yes. Let $f(x) = 1$ for $x \geq 0$ and $f(x) = -1$ for $x < 0$. Then $f(x) \cdot f(x) = 1$ for all x, so $f^2 \in K^*$ but f is not in K^*. Thus fK^* has order 2 in F^*/K^*.
25. U
27. The multiplicative group U of complex numbers of absolute value 1
29. Let $G = \mathbb{Z}_2 \times \mathbb{Z}_4$. Then $H = \langle(1, 0)\rangle$ is isomorphic to $K = \langle(0, 2)\rangle$, but G/H is isomorphic to $\mathbb{Z}_4$ while G/K is isomorphic to $\mathbb{Z}_2 \times \mathbb{Z}_2$.
31. **a.** $\{e\}$ **b.** The whole group

SECTION 16

1. $X_{\rho_0} = X$, $X_{\rho_1} = \{C\}$, $X_{\rho_2} = \{m_1, m_2, d_1, d_2, C\}$, $X_{\rho_3} = \{C\}$,
$X_{\mu_1} = \{s_1, s_3, m_1, m_2, C, P_1, P_3\}$, $X_{\mu_2} = \{s_2, s_4, m_1, m_2, C, P_2, P_4\}$,
$X_{\delta_1} = \{2, 4, d_1, d_2, C\}$, $X_{\delta_2} = \{1, 3, d_1, d_2, C\}$.
3. $\{1, 2, 3, 4\}$, $\{s_1, s_2, s_3, s_4\}$, $\{m_1, m_2\}$, $\{d_1, d_2\}$, $\{C\}$, $\{P_1, P_2, P_3, P_4\}$
7. A transitive G-set has just one orbit.
9. **a.** $\{s_1, s_2, s_3, s_4\}$ and $\{P_1, P_2, P_3, P_4\}$
13. **b.** The set of points on the circle with center at the origin and passing through P
c. The cyclic subgroup $\langle 2\pi \rangle$ of $G = \mathbb{R}$
17. **a.** $K = g_0 H g_0^{-1}$.
b. *Conjecture:* H and K should be conjugate subgroups of G.