19.

	X	Y		Z		
	a	a	b	a	b	c
0	a	a	b	a	b	c
1	a	b	a	b	c	a
2	a	a	b	c	a	b
3	a	b	a	a	b	c
4	a	a	b	b	c	a
5	a	b	a	c	a	b

There are four of them: X, Y, Z, and $\mathbb{Z}_6$.

SECTION 17

1. 5 **3.** 2 **5.** 11,712
7. **a.** 45 **b.** 231
9. **a.** 90 **b.** 6,246

SECTION 18

1. 0 **3.** 1 **5.** (1, 6)
7. Commutative ring, no unity, not a field
9. Commutative ring with unity, not a field
11. Commutative ring with unity, not a field
13. No. $\{ri \mid r \in \mathbb{R}\}$ is not closed under multiplication.
15. $(1, 1), (1, -1), (-1, 1), (-1, -1)$
17. All nonzero $q \in \mathbb{Q}$ **19.** 1, 3
21. Let $\mathbb{R} = \mathbb{Z}$ with unity 1 and $\mathrm{R}' = \mathbb{Z} \times \mathbb{Z}$ with unity $1' = (1, 1)$. Let $\phi : R \to R'$ be defined by $\phi(n) = (n, 0)$. Then $\phi(1) = (1, 0) \neq 1'$.
23. $\phi_1 : \mathbb{Z} \to \mathbb{Z}$ where $\phi_1(n) = 0$, $\phi_2 : \mathbb{Z} \to \mathbb{Z}$ where $\phi_2(n) = n$
25. $\phi_1 : \mathbb{Z} \times \mathbb{Z} \to \mathbb{Z}$ where $\phi_1(n, m) = 0$, $\phi_2 : \mathbb{Z} \times \mathbb{Z} \to \mathbb{Z}$ where $\phi_2(n, m) = n$
$\phi_3 : \mathbb{Z} \times \mathbb{Z} \to \mathbb{Z}$ where $\phi_3(n, m) = m$
27. The reasoning is not correct since a product $(X - I_3)(X + I_3)$ of two matrices may be the zero matrix 0 without having either matrix be 0. Counterexample:

$$\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}^2 = I_3.$$

31. $a = 2, b = 3$ in $\mathbb{Z}_6$
33. **a.** T **c.** F **e.** T **g.** T **i.** T

SECTION 19

1. 0, 3, 5, 8, 9, 11 **3.** No solutions **5.** 0 **7.** 0 **9.** 12
11. $a^4 + 2a^2b^2 + b^4$ **13.** $a^6 + 2a^3b^3 + b^6$
17. **a.** F **c.** F **e.** T **g.** F **i.** F