19. **1.** $\text{Det}(A) = 0$. **2.** The column vectors of A are dependent.
3. The row vectors of A are dependent. **4.** Zero is an eigenvalue of A.
5. A is not invertible.

SECTION 20

1. 3 or 5 **3.** Any of 3, 5, 6, 7, 10, 11, 12, or 14. **5.** 2

7.

$\varphi(1) = 1$	$\varphi(7) = 6$	$\varphi(13) = 12$	$\varphi(19) = 18$	$\varphi(25) = 20$
$\varphi(2) = 1$	$\varphi(8) = 4$	$\varphi(14) = 6$	$\varphi(20) = 8$	$\varphi(26) = 12$
$\varphi(3) = 2$	$\varphi(9) = 6$	$\varphi(15) = 8$	$\varphi(21) = 12$	$\varphi(27) = 18$
$\varphi(4) = 2$	$\varphi(10) = 4$	$\varphi(16) = 8$	$\varphi(22) = 10$	$\varphi(28) = 12$
$\varphi(5) = 4$	$\varphi(11) = 10$	$\varphi(17) = 16$	$\varphi(23) = 22$	$\varphi(29) = 28$
$\varphi(6) = 2$	$\varphi(12) = 4$	$\varphi(18) = 6$	$\varphi(24) = 8$	$\varphi(30) = 8$

9. $(p-1)(q-1)$ **11.** $1 + 4\mathbb{Z}, 3 + 4\mathbb{Z}$ **13.** No solutions
15. No solutions
17. $3 + 65\mathbb{Z}, 16 + 65\mathbb{Z}, 29 + 65\mathbb{Z}, 42 + 65\mathbb{Z}, 55 + 65\mathbb{Z}$
19. 1 **21.** 9
23. **a.** F **c.** T **e.** T **g.** F **i.** F

SECTION 21

1. $\{q_1 + q_2 i \mid q_1, q_2 \in \mathbb{Q}\}$
15. It is isomorphic to the ring D of all rational numbers that can be expressed as a quotient of integers with denominator some power of 2.
17. It runs into trouble when we try to prove the transitive property in the proof of Lemma 5.4.2, for multiplicative cancellation may not hold. For $R = \mathbb{Z}_6$ and $T = \{1, 2, 4\}$ we have $(1, 2) \sim (2, 4)$ since $(1)(4) = (2)(2) = 4$ and $(2, 4) \sim (2, 1)$ since $(2)(1) = (4)(2)$ in $\mathbb{Z}_6$. However, $(1, 2)$ is not equivalent to $(2, 1)$ because $(1)(1) \neq (2)(2)$ in $\mathbb{Z}_6$.

SECTION 22

1. $f(x) + g(x) = 2x^2 + 5$, $f(x)g(x) = 6x^2 + 4x + 6$
3. $f(x) + g(x) = 5x^2 + 5x + 1$, $f(x)g(x) = x^3 + 5x$
5. 16 **7.** 7 **9.** 2 **11.** 0 **13.** 2, 3 **15.** 0, 2, 4
17. 0, 1, 2, 3
21. $0, x - 5, 2x - 10, x^2 - 25, x^2 - 5x, x^4 - 5x^3$. (Other answers are possible.)
23. **a.** T **c.** T **e.** F **g.** T **i.** T
25. **a.** They are the units of D. **b.** $1, -1$ **c.** 1, 2, 3, 4, 5, 6
27. **b.** F **c.** $F[x]$ **31.** **a.** 4, 27 **b.** $\mathbb{Z}_2 \times \mathbb{Z}_2$, $\mathbb{Z}_3 \times \mathbb{Z}_3 \times \mathbb{Z}_3$

SECTION 23

1. $q(x) = x^4 + x^3 + x^2 + x - 2, r(x) = 4x + 3$
3. $q(x) = 6x^4 + 7x^3 + 2x^2 - x + 2, r(x) = 4$
5. 2, 3 **7.** 3, 10, 5, 11, 14, 7, 12, 6
9. $(x-1)(x+1)(x-2)(x+2)$
11. $(x-3)(x+3)(2x+3)$