13. Yes. It is of degree 3 with no zeros in $\mathbb{Z}_5$.
$2x^3 + x^2 + 2x + 2$
15. *Partial answer:* $g(x)$ is irreducible over $\mathbb{R}$, but it is not irreducible over $\mathbb{C}$.
19. Yes. $p = 3$ **21.** Yes. $p = 5$
25. **a.** T **c.** T **e.** T **g.** T **i.** T
27. $x^2 + x + 1$
29. $x^2 + 1, x^2 + x + 2, x^2 + 2x + 2, 2x^2 + 2, 2x^2 + x + 1, 2x^2 + 2x + 1$
31. $p(p-1)^2/2$

SECTION 24

1. $1e + 0a + 3b$ **3.** $2e + 2a + 2b$ **5.** j **7.** $(1/50)j - (3/50)k$
9. $\mathbb{R}^*$, that is, $\{a_1 + 0i + 0j + 0k \mid a_1 \in \mathbb{R}, a_1 \neq 0\}$
11. **a.** F **c.** F **e.** F **g.** T **i.** T
c. If $|A| = 1$, then $\text{End}(A) = \{0\}$. **e.** $0 \in \text{End}(A)$ is not in $\text{Iso}(A)$.
19. **a.** $K = \begin{bmatrix} i & 0 \\ 0 & -i \end{bmatrix}$.
b. Denoting by B the matrix with coefficient b and by C the matrix with coefficient c and the 2×2 identity matrix by I, we must check that

$$B^2 = -I, C^2 = -I, K^2 = -I,$$
$$CK = B, KB = C, CB = -K, KC = -B, \text{ and } BK = -C.$$

c. We should check that ϕ is one to one.

SECTION 25

1. $a < x < x^2 < x^3 < \cdots < x^n \cdots$ for any $a \in R$.
3. $m + n\sqrt{2}$ is positive if $m > 0$ and $n < 0$, or if $m > 0$ and $m^2 > 2n^2$, or if $n < 0$ and $2n^2 > m^2$.
5. **i.** a c e d b **ii.** e c b a d
7. **i.** d a b c e **ii.** d c e a b
9. **i.** c a e d b **ii.** e c b a d
11. d b a e c **13.** d e b a c
15. **a.** T **c.** F **e.** T **g.** T **i.** F

SECTION 26

1. There are just nine possibilities:
$\phi(1,0) = (1,0)$ while $\phi(0,1) = (0,0)$ or $(0,1)$,
$\phi(1,0) = (0,1)$ while $\phi(0,1) = (0,0)$ or $(1,0)$,
$\phi(1,0) = (1,1)$ while $\phi(0,1) = (0,0)$, and
$\phi(1,0) = (0,0)$ while $\phi(0,1) = (0,0), (1,0), (0,1)$, or $(1,1)$.
3. $\langle 0 \rangle = \{0\}$, $\mathbb{Z}_{12}/\{0\} \simeq \mathbb{Z}_{12}$
$\langle 1 \rangle = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11\}$, $\mathbb{Z}_{12}/\langle 1 \rangle \simeq \{0\}$
$\langle 2 \rangle = \{0, 2, 4, 6, 8, 10\}$, $\mathbb{Z}_{12}/\langle 2 \rangle \simeq \mathbb{Z}_2$
$\langle 3 \rangle = \{0, 3, 6, 9\}$, $\mathbb{Z}_{12}/\langle 3 \rangle \simeq \mathbb{Z}_3$
$\langle 4 \rangle = \{0, 4, 8\}$, $\mathbb{Z}_{12}/\langle 4 \rangle \simeq \mathbb{Z}_4$
$\langle 6 \rangle = \{0, 6\}$, $\mathbb{Z}_{12}/\langle 6 \rangle \simeq \mathbb{Z}_6$
9. Let $\phi : \mathbb{Z} \to \mathbb{Z} \times \mathbb{Z}$ be given by $\phi(n) = (n, 0)$ for $n \in \mathbb{Z}$.