11. R/R and $R/\{0\}$ are not of real interest because R/R is the ring containing only the zero element, and $R/\{0\}$ is isomorphic to R.
13. $\mathbb{Z}$ is an integral domain. $\mathbb{Z}/4\mathbb{Z}$ is isomorphic to $\mathbb{Z}_4$, which has a divisor 2 of 0.
15. $\{(n, n) \mid n \in \mathbb{Z}\}$. (Other answers are possible.)
31. The nilradical of $\mathbb{Z}_{12}$ is $\{0, 6\}$. The nilradical of $\mathbb{Z}$ is $\{0\}$ and the nilradical of $\mathbb{Z}_{32}$ is $\{0, 2, 4, 6, 8, \cdots, 30\}$.
35. **a.** Let $R = \mathbb{Z}$ and let $N = 4\mathbb{Z}$. Then $\sqrt{N} = 2\mathbb{Z} \neq 4\mathbb{Z}$
b. Let $R = \mathbb{Z}$ and let $N = 2\mathbb{Z}$. Then $\sqrt{N} = N$.

SECTION 27

1. $\{0, 2, 4\}$ and $\{0, 3\}$ are both prime and maximal.
3. $\{(0, 0), (1, 0)\}$ and $\{(0, 0), (0, 1)\}$ are both prime and maximal.
5. 1 **7.** 2 **9.** 1, 4 **15.** $2\mathbb{Z} \times \mathbb{Z}$ **17.** $4\mathbb{Z} \times \{0\}$
19. Yes. $x^2 - 6x + 6$ is irreducible over $\mathbb{Q}$ by Eisenstein with $p = 2$.
21. Yes. $\mathbb{Z}_2 \times \mathbb{Z}_3$
23. No. Enlarging the domain to a field of quotients, you would have to have a field containing two different prime fields $\mathbb{Z}_p$ and $\mathbb{Z}_q$, which is impossible.

SECTION 28

1. $-3x^3 + 7x^2y^2z - 5x^2yz^3 + 2xy^3z^5$
3. $2x^2yz^2 - 2xy^2z^2 - 7x + 3y + 10z^3$
5. $2z^5y^3x - 5z^3yx^2 + 7zy^2x^2 - 3x^3$
7. $10z^3 - 2z^2y^2x + 2z^2yx^2 + 3y - 7x$
9. $1 < z < y < x < z^2 < yz < y^2 < xz < xy < x^2 < z^3 < yz^2 < y^2z < y^3 < xz^2 < xyz < xy^2 < x^2z < x^2y < x^3 < \cdots$
11. $3y^2z^5 - 8z^7 + 5y^3z^3 - 4x$ **13.** $3yz^3 - 8xy - 4xz + 2yz + 38$
15. $\langle y^5 + y^3, y^3 + z, x - y^4 \rangle$ **17.** $\langle y^2z^3 + 3, -3y - 2z, y^2z^2 + 3 \rangle$
19. $\{1\}$ **21.** $\{x - 1\}$
23. $\{2x + y - 5, y^2 - 9y + 18\}$
The algebraic variety is $\{(1, 3), (-\frac{1}{2}, 6)\}$.
25. $\{x + y, y^3 - y + 1\}$
The algebraic variety consists of one point $(a, -a)$ where $a \approx 1.3247$.
27. **a.** *T* **c.** *T* **e.** *T* **g.** *T* **i.** *F*

SECTION 29

1. $x^2 - 2x - 1$ **3.** $x^2 - 2x + 2$
5. $x^{12} + 3x^8 - 4x^6 + 3x^4 + 12x^2 + 5$
7. $\mathrm{Irr}(\alpha, \mathbb{Q}) = x^4 - \frac{2}{3}x^2 - \frac{62}{9}$; $\deg(\alpha, \mathbb{Q}) = 4$
9. Algebraic, $\deg(\alpha, F) = 2$
11. Transcendental
13. Algebraic, $\deg(\alpha, F) = 2$
15. Algebraic, $\deg(\alpha, F) = 1$
17. $x^2 + x + 1 = (x - \alpha)(x + 1 + \alpha)$
23. **a.** *T* **c.** *T* **e.** *F* **g.** *F* **i.** *F*
25. **b.** $x^3 + x^2 + 1 = (x - \alpha)(x - \alpha^2)[x - (1 + \alpha + \alpha^2)]$
27. It is the monic polynomial in $F[x]$ of *minimal* degree having α as a zero.