

## SECTION 30

1.  $\{(0, 1), (1, 0)\}, \{(1, 1), (-1, 1)\}, \{(2, 1), (1, 2)\}$ . (Other answers are possible.)  
 3. No.  $2(-1, 1, 2) - 4(2, -3, 1) + (10, -14, 0) = (0, 0, 0)$   
 5.  $\{1\}$       7.  $\{1, i\}$       9.  $\{1, \sqrt[3]{2}, \sqrt{2}, (\sqrt[3]{2})^3\}$   
 15. a.  $T$       c.  $T$       e.  $F$       g.  $F$       i.  $T$   
 17. a. The subspace of  $V$  generated by  $S$  is the intersection of all subspaces of  $V$  containing  $S$ .  
 19. *Partial answer:* A basis for  $F^n$  is

$$\{(1, 0, \dots, 0), (0, 1, \dots, 0), \dots, (0, 0, \dots, 1)\}$$

where 1 is the multiplicative identity of  $F$ .

25. a. A homomorphism  
 b. *Partial answer:* The kernel (or nullspace) of  $\phi$  is  $\{\alpha \in V \mid \phi(\alpha) = 0\}$ .  
 c.  $\phi$  is an isomorphism of  $V$  with  $V'$  if  $\text{Ker}(\phi) = \{0\}$  and  $\phi$  maps  $V$  onto  $V'$ .

## SECTION 31

1.  $2, \{1, \sqrt{2}\}$       3.  $4, \{1, \sqrt{3}, \sqrt{2}, \sqrt{6}\}$   
 5.  $6, \{1, \sqrt{2}, \sqrt[3]{2}, \sqrt{2}(\sqrt[3]{2}), (\sqrt[3]{2})^2, \sqrt{2}(\sqrt[3]{2})^2\}$       7.  $2, \{1, \sqrt{6}\}$   
 9.  $9, \{1, \sqrt[3]{2}, \sqrt[3]{4}, \sqrt[3]{3}, \sqrt[3]{6}, \sqrt[3]{12}, \sqrt[3]{9}, \sqrt[3]{18}, \sqrt[3]{36}\}$   
 11.  $2, \{1, \sqrt{2}\}$       13.  $2, \{1, \sqrt{2}\}$   
 19. a.  $F$       c.  $F$       e.  $F$       g.  $F$       i.  $F$   
 23. *Partial answer:* Extensions of degree  $2^n$  for  $n \in \mathbb{Z}^+$  are obtained.

## SECTION 32

All odd-numbered answers require proofs and are not listed here.

## SECTION 33

1. Yes      3. Yes      5. 6      7. 0

## SECTION 34

1. a.  $K = \{0, 3, 6, 9\}$ .  
 b.  $0 + K = \{0, 3, 6, 9\}$ ,  $1 + K = \{1, 4, 7, 10\}$ ,  $2 + K = \{2, 5, 8, 11\}$ .  
 c.  $\mu(0 + K) = 0$ ,  $\mu(1 + K) = 2$ ,  $\mu(2 + K) = 1$ .  
 3. a.  $HN = \{0, 2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22\}$ ,  $H \cap N = \{0, 12\}$ .  
 b.  $0 + N = \{0, 6, 12, 18\}$ ,  $2 + N = \{2, 8, 14, 20\}$ ,  $4 + N = \{4, 10, 16, 22\}$ .  
 c.  $0 + (H \cap N) = \{0, 12\}$ ,  $4 + (H \cap N) = \{4, 16\}$ ,  $8 + (H \cap N) = \{8, 20\}$ .  
 d.  $\phi(0 + N) = 0 + (H \cap N)$ ,  $\phi(2 + N) = 8 + (H \cap N)$ ,  $\phi(4 + N) = 4 + (H \cap N)$ .  
 5. a.  $0 + H = \{0, 4, 8, 12, 16, 20\}$ ,  $1 + H = \{1, 5, 9, 13, 17, 21\}$ ,  
 $2 + H = \{2, 6, 10, 14, 18, 22\}$ ,  $3 + H = \{3, 7, 11, 15, 19, 23\}$ .  
 b.  $0 + K = \{0, 8, 16\}$ ,  $1 + K = \{1, 9, 17\}$ ,  $2 + K = \{2, 10, 18\}$ ,  
 $3 + K = \{3, 11, 19\}$ ,  
 $4 + K = \{4, 12, 20\}$ ,  $5 + K = \{5, 13, 21\}$ ,  $6 + K = \{6, 14, 22\}$ ,  
 $7 + K = \{7, 15, 23\}$ .