c. $0 + K = \{0, 8, 16\}, 4 + K = \{4, 12, 20\}$.
d. $(0 + K) + (H/K) = H/K = \{0 + K, 4 + K\} = \{\{0, 8, 16\}, 4, 12, 20\}\}$
$(1 + K) + (H/K) = \{1 + K, 5 + K\} = \{\{1, 9, 17\}, \{5, 13, 21\}\}$
$(2 + K) + (H/K) = \{2 + K, 6 + K\} = \{\{2, 10, 18\}, \{6, 14, 22\}\}$
$(3 + K) + (H/K) = \{3 + K, 7 + K\} = \{\{3, 11, 19\}, \{7, 15, 23\}\}$.
e. $\phi(0 + H) = (0 + K) + (H/K), \phi(1 + H) = (1 + K) + (H/K)$,
$\phi(2 + H) = (2 + K) + (H/K), \phi(3 + H) = (3 + K) + (H/K)$.

SECTION 35

1. The refinements $\{0\} < 250\mathbb{Z} < 10\mathbb{Z} < \mathbb{Z}$ of $\{0\} < 10\mathbb{Z} < \mathbb{Z}$ and $\{0\} < 250\mathbb{Z} < 25\mathbb{Z} < \mathbb{Z}$ of $0 < 25\mathbb{Z} < \mathbb{Z}$ are isomorphic.
3. The given series are isomorphic.
5. The refinements
$\{(0, 0)\} < (4800\mathbb{Z}) \times \mathbb{Z} < (240\mathbb{Z}) \times \mathbb{Z} < (60\mathbb{Z}) \times \mathbb{Z} < (10\mathbb{Z}) \times \mathbb{Z} < \mathbb{Z} \times \mathbb{Z}$ of the first series and
$\{(0, 0)\} < \mathbb{Z} \times (4800\mathbb{Z}) < \mathbb{Z} \times (480\mathbb{Z}) < \mathbb{Z} \times (80\mathbb{Z}) < \mathbb{Z} \times (20\mathbb{Z}) < \mathbb{Z} \times \mathbb{Z}$ of the second series are isomorphic refinements.
7. $\{0\} < \langle 16\rangle < \langle 8\rangle < \langle 4\rangle < \langle 2\rangle < \mathbb{Z}_{48}$
$\{0\} < \langle 24\rangle < \langle 8\rangle < \langle 4\rangle < \langle 2\rangle < \mathbb{Z}_{48}$
$\{0\} < \langle 24\rangle < \langle 12\rangle < \langle 4\rangle < \langle 2\rangle < \mathbb{Z}_{48}$
$\{0\} < \langle 24\rangle < \langle 12\rangle < \langle 6\rangle < \langle 2\rangle < \mathbb{Z}_{48}$
$\{0\} < \langle 24\rangle < \langle 12\rangle < \langle 6\rangle < \langle 3\rangle < \mathbb{Z}_{48}$
9. $\{(\rho_0, 0)\} < A_3 \times \{0\} < S_3 \times (0) < S_3 \times \mathbb{Z}_2$
$\{(\rho_0, 0)\} < \{\rho_0\} \times \mathbb{Z}_2 < A_3 \times \mathbb{Z}_2 < S_3 \times \mathbb{Z}_2$
$\{(\rho_0, 0)\} < A_3 \times \{0\} < A_3 \times \mathbb{Z}_2 < S_3 \times \mathbb{Z}_2$
11. $\{\rho_0\} \times \mathbb{Z}_4$ **13.** $\{\rho_0\} \times \mathbb{Z}_4 \leq \{\rho_0\} \times \mathbb{Z}_4 \leq \{\rho_0\} \times \mathbb{Z}_4 \leq \cdots$.
17. **a.** *T* **c.** *T* **e.** *F* **g.** *F* **i.** *T*
i. The Jordan-Hölder theorem applied to the groups $\mathbb{Z}_n$ implies the Fundamental Theorem of Arithmetic.
19. Yes. $\{\rho_0\} < \{\rho_0, \rho_2\} < \{\rho_0, \rho_1, \rho_2, \rho_3\} < D_4$ is a composition (actually a principal) series and all factor groups are isomorphic to $\mathbb{Z}_2$ and are thus abelian.
21. *Chain (3)*
$\{0\} \leq \langle 12\rangle \leq \langle 12\rangle \leq \langle 12\rangle$
$\leq \langle 12\rangle \leq \langle 12\rangle \leq \langle 4\rangle$
$\leq \langle 2\rangle \leq \mathbb{Z}_{24} \leq \mathbb{Z}_{24}$

Chain (4)
$\{0\} \leq \langle 12\rangle < \langle 12\rangle \leq \langle 6\rangle$
$\leq \langle 6\rangle \leq \langle 6\rangle \leq \langle 3\rangle$
$\leq \langle 3\rangle \leq \mathbb{Z}_{24} \leq \mathbb{Z}_{24}$

Isomorphisms

$\langle 12\rangle/\{0\} \simeq \langle 12\rangle/\{0\} \simeq \mathbb{Z}_2$, $\langle 12\rangle/\langle 12\rangle \simeq \langle 6\rangle/\langle 6\rangle \simeq \{0\}$,
$\langle 12\rangle/\langle 12\rangle \simeq \langle 3\rangle/\langle 3\rangle \simeq \{0\}$, $\langle 12\rangle/\langle 12\rangle \simeq \langle 12\rangle/\langle 12\rangle \simeq \{0\}$,
$\langle 12\rangle/\langle 12\rangle \simeq \langle 6\rangle/\langle 6\rangle \simeq \{0\}$, $\langle 4\rangle/\langle 12\rangle \simeq \mathbb{Z}_{24}/\langle 3\rangle \simeq \mathbb{Z}_3$
$\langle 2\rangle/\langle 4\rangle \simeq \langle 6\rangle/\langle 12\rangle \simeq \mathbb{Z}_2$, $\mathbb{Z}_{24}/\langle 2\rangle \simeq \langle 3\rangle/\langle 6\rangle \simeq \mathbb{Z}_2$
$\mathbb{Z}_{24}/\mathbb{Z}_{24} \simeq \mathbb{Z}_{24}/\mathbb{Z}_{24} \simeq \{0\}$

SECTION 36

1. 3 **3.** 1, 3
5. The Sylow 3-subgroups are $\langle(1, 2, 3)\rangle$, $\langle(1, 2, 4)\rangle$, $\langle(1, 3, 4)\rangle$, and $\langle(2, 3, 4)\rangle$. Also $(3, 4)\langle(1, 2, 3)\rangle(3, 4) = \langle(1, 2, 4)\rangle$, etc.