a^2	a	1	a^3
a^3	a^2	a	1
1	a^3	a^2	a
a	1	a^3	a^2

5. $\mathbb{Z}_{21}$. $(a, b : a^7 = 1, b^3 = 1, ba = a^2 b)$

SECTION 41

1. **a.** $2P_1P_3 - 3P_1P_4 + P_1P_6 - 3P_2P_3 + 3P_2P_4 - 5P_3P_4 + 4P_3P_6 - 5P_4P_6$
b. No **c.** Yes

3. $C_i(P) = Z_i(P) = B_i(P) = H_i(P) = 0$ for $i > 0$. $B_0(P) = 0$. $Z_0(P) \simeq \mathbb{Z}$ and is generated by the 0-cycle P. $H_0(P) \simeq \mathbb{Z}$.

5. $C_i(X) = Z_i(X) = B_i(X) = H_i(X) = 0$ for $i > 0$. $B_0(X) \simeq \mathbb{Z}$ and is generated by the 0-chain $P_2 - P_1$. $Z_0(X) \simeq \mathbb{Z} \times \mathbb{Z}$ and is generated by the two 0-cycles P_1 and P_2. Since $Z_0(X)/B_0(X)$ "indentifies P_1 with P_2," $H_0(X) \simeq \mathbb{Z}$ and is generated by the coset $P_1 + B_0(X)$.

7. **a.** An **oriented n-simplex** is an ordered sequance $P_1P_2 \cdots P_{n+1}$.
b. The **boundary of** $P_1P_2 \cdots P_{n+1}$ is given by

$$\partial_n(P_1P_2 \cdots P_{n+1}) \sum_{i=1}^{n+1} (-1)^{i+1} P_1P_2 \cdots P_{i-1}P_{i+1} \cdots P_{n+1}.$$

c. Each individual *summand* of the boundary of an oriented n-simplex is a **face of the simplex.**

11. **a.** $\delta^{(n)}\left(\sum_i m_i\sigma_i\right) = \sum_i m_i\delta^{(n)}(\sigma_i)$

13. $H^{(n)}(X) = Z^{(n)}(X)/B^{(n)}(X)$
$H^{(0)}(S) \simeq \mathbb{Z}$ and is generated by $(P_1 + P_2 + P_3 + P_4) + \{0\}$
$H^{(1)}(S) = 0$
$H^{(2)}(S) \simeq \mathbb{Z}$ and is generated by $P_1P_2P_3 + B^{(2)}(S)$

SECTION 42

1. $H_0(X) \simeq \mathbb{Z}$. $H_1(X) \simeq \mathbb{Z} \times \mathbb{Z}$. $H_n(X) = 0$ for $n > 1$.
3. $H_0(X) \simeq \mathbb{Z} \times \mathbb{Z}$. $H_1(X) \simeq \mathbb{Z}$. $H_2(X) \simeq \mathbb{Z}$. $H_n(X) = 0$ for $n > 2$.
5. $H_0(X) \simeq \mathbb{Z}$. $H_1(X) \simeq \mathbb{Z}$. $H_2(X) \simeq \mathbb{Z}$. $H_n(X) = 0$ for $n > 2$.
7. **a.** T **c.** F **e.** T **g.** T **i.** F
9. $H_0(X) \simeq \mathbb{Z}$. $H_1(X) \simeq \mathbb{Z} \times \mathbb{Z} \times \mathbb{Z}$. $H_2(X) \simeq \mathbb{Z} \times \mathbb{Z}$. $H_n(X) = 0$ for $n > 2$.
11. $H_0(X) \simeq \mathbb{Z}$. $H_1(X) \simeq \mathbb{Z} \times \mathbb{Z} \times \mathbb{Z} \times \mathbb{Z}$. $H_2(X) \simeq \mathbb{Z}$. $H_n(X) = 0$ for $n > 2$.

SECTION 43

1. Both counts show that $\chi(X) = 1$.
3. It will hold for a square region, for such a region is homeomorphic to E^2. It obviously does not hold for two disjoint 2-cells, for each can be mapped continuously onto the other, and such a map has no fixed points.
5. $H_0(X) \simeq \mathbb{Z} \times \mathbb{Z}$. $H_1(X) \simeq \mathbb{Z} \times \mathbb{Z} \times \mathbb{Z}_2 \times \mathbb{Z}_2$. $H_n(X) = 0$ for $n > 1$.
7. $2 - 2n$