9. $H_0(X) \simeq \mathbb{Z}$. $H_1(X) \simeq \underbrace{\mathbb{Z} \times \mathbb{Z} \times \cdots \times \mathbb{Z}}_{(q-1)\text{ factors}} \times \mathbb{Z}_2$. $H_n(X) = 0$ for $n > 1$.

11. Let Q be a vertex of b, and let c be the 2-chain consisting of all 2-simplexes of X, all oriented the same way, so that $c \in Z_2(X)$.

a. f_{*0} is given by $f_{*0}(Q + B_0(X)) = Q + B_0(b)$.
f_{*1} is given by $f_{*1}((ma + nb) + B_1(X)) = nb + B_1(b)$.
f_{*2} is given by $f_{*2}(c + B_1(X)) = 0$.

b. f_{*0} is as in (a).
f_{*1} is given by $f_{*1}((ma + nb) + B_1(X)) = 2nb + B_1(b)$.
f_{*2} is as in (a).

13. Let Q be a vertex on b.
f_{*0} is given by $f_{*0}(Q + B_0(X)) = Q + B_0(b)$.
f_{*1} is given by $f_{*1}((ma + nb) + B_1(X)) = nb + B_1(b)$, where $m = 0, 1$.
f_{*2} is trivial, since both $H_2(X)$ and $H_2(b)$ are 0.

SECTION 44

5. For Theorem 44.4, the condition $f_{k-1}\partial_k = \partial'_k f_k$ implies that

$$f_{k-1}(B_{k-1}(A)) \subseteq B_{k-1}(A').$$

Then Exercise 14.39 shows that f_{k-1} induces a natural homomorphism of $Z_{k-1}(A)/B_{k-1}(A)$ into $Z_{k-1}(A')/B_{k-1}(A')$. This is the correct way to view Theorem 44.4.
For Theorem 44.7, if we use Exercise 14.39, the fact that $\partial_k(A'_k) \subseteq A'_{k-1}$ shows that ∂_k induces a natural homomorphism $\bar{\partial}_k : (A_k/A'_k) \to (A_{k-1}/A'_{k-1})$.

7. The exact homology sequence is

$$\begin{aligned}[H_2(a) = 0] &\xrightarrow{i_{*2}} [H_2(X) \simeq \mathbb{Z}] \xrightarrow{j_{*2}} [H_2(X, a) \simeq \mathbb{Z}] \xrightarrow{\partial_{*2}} [H_1(a) \simeq \mathbb{Z}] \\ &\xrightarrow{i_{*1}} [H_1(X) \simeq \mathbb{Z} \times \mathbb{Z}] \xrightarrow{j_{*1}} [H_1(X, a) \simeq \mathbb{Z}] \xrightarrow{\partial_{*1}} [H_0(a) \simeq \mathbb{Z}] \\ &\xrightarrow{i_{*0}} [H_0(X) \simeq \mathbb{Z}] \xrightarrow{j_{*0}} [H_0(X, a) = 0].\end{aligned}$$

j_{*2} maps a generator $c + B_2(X)$ of $H_2(X)$ onto the generator

$$(c + C_2(a)) + B_2(X, a)$$

of $H_2(X, a)$ and is an isomorphism. Thus (kernel j_{*2}) = (image i_{*2}) = 0.
∂_{*2} maps everything onto 0, so (kernal ∂_{*2}) = (image j_{*2}) $\simeq \mathbb{Z}$.
i_{*1} maps the generator $a + B_1(a)$ onto $(a + 0b) + B_1(X)$, so i_{*1} is an isomorphism *into*, and (kernal i_{*1}) = (image ∂_{*2}) = 0.
j_{*1} maps $(ma + nb) + B_1(X)$ onto $(nb + C_1(a)) + B_1(X, a)$, so (kernal j_{*1}) = (image i_{*1}) $\simeq \mathbb{Z}$.
∂_{*1} maps $(nb + C_1(a)) + B_1(X, a)$ onto 0, so (kernal ∂_{*1}) = (image j_{*1}) $\simeq \mathbb{Z}$.
For a vertex Q of a, i_{*0} maps $Q + B_0(a)$ onto $Q + B_0(X)$, so i_{*0} is an isomorphism, and (kernal i_{*0}) = (image ∂_{*1}) = 0.
j_{*0} maps $Q + B_0(X)$ onto $B_0(X, a)$ in $H_0(X, a)$, so (kernal j_{*0}) = (image i_{*0}) $\simeq \mathbb{Z}$.

9. The answer is formally identical with that in Exercise 44.7.

11. *Partial answer:* The exact homology sequence is

$$\begin{aligned}[H_2(Y) = 0] &\xrightarrow{i_{*2}} [H_2(X) = 0] \xrightarrow{j_{*2}} [H_2(X, Y) \simeq \mathbb{Z}] \xrightarrow{\partial_{*2}} [H_1(Y) \simeq \mathbb{Z} \times \mathbb{Z}] \\ &\xrightarrow{i_{*1}} [H_1(X) \simeq \mathbb{Z}] \xrightarrow{j_{*1}} [H_1(X, Y) \simeq \mathbb{Z}] \xrightarrow{\partial_{*1}} [H_0(Y) \simeq \mathbb{Z} \times \mathbb{Z}] \\ &\xrightarrow{i_{*0}} [H_0(X) \simeq \mathbb{Z}] \xrightarrow{j_{*0}} [H_0(X, Y) = 0].\end{aligned}$$