The verification of exactness is left to you. Note that the edge P_1Q_1 of Fig. 42.11 gives rise to a generator of $H_1(X, Y)$. Starting with ∂_{*2}, these maps are very interesting.

SECTION 45

1. Yes **3.** No **5.** No. **7.** Yes

9. In $\mathbb{Z}[x]$: only $2x-7, -2x+7$

In $\mathbb{Q}[x]: 4x-14, x-\frac{7}{2}, 6x-21, -8x+28$

In $\mathbb{Z}_{11}[x]: 2x-7, 10x-2, 6x+1, 3x-5, 5x-1$

11. $26, -26$ **13.** $198, -198$

15. It is already "primitive" because every nonzero element of $\mathbb{Q}$ is a unit. Indeed $18ax^2-12ax+48a$ is primitive for all $a \in \mathbb{Q}, a \neq 0$.

17. $2ax^2-3ax+6a$ is primitive for all $a \neq 0$ in $\mathbb{Z}_7$ because every such element a is a unit in $\mathbb{Z}_7$.

21. **a.** *T* **c.** *T* **e.** *T* **g.** *F* **i.** *F*

i. Either p or one of its associates must appear in every factorization *into irreducibles.*

23. $2x+4$ is irreducible in $\mathbb{Q}[x]$ but not in $\mathbb{Z}[x]$.

31. *Partial answer:* $x^3-y^3=(x-y)(x^2+xy+y^2)$

SECTION 46

1. Yes **3.** No. (1) is violated. **5.** Yes

7. 61 **9.** x^3+2x-1 **11.** 66

13. **a.** *T* **c.** *T* **e.** *T* **g.** *T* **i.** *T*

23. *Partial answer:* The equation $ax=b$ has a solution in $\mathbb{Z}_n$ for nonzero $a, b \in \mathbb{Z}_n$ if and only if the positive gcd of a and n in $\mathbb{Z}$ divides b.

SECTION 47

1. $5=(1+2i)(1-2i)$ **3.** $4+3i=(1+2i)(2-i)$

5. $6=(2)(3)=(-1+\sqrt{-5})(-1-\sqrt{-5})$ **7.** $7-i$

15. **c. i)** order 9, characteristic 3 **ii)** order 2, characteristic 2

iii) order 5, characteristic 5

SECTION 48

1. $\sqrt{2}, -\sqrt{2}$ **3.** $3+\sqrt{2}, 3-\sqrt{2}$ **5.** $\sqrt{2}+i, \sqrt{2}-i, -\sqrt{2}+i, -\sqrt{2}-i$

7. $\sqrt{1+\sqrt{2}}, -\sqrt{1+\sqrt{2}}, \sqrt{1-\sqrt{2}}, -\sqrt{1-\sqrt{2}}$ **9.** $\sqrt{3}$

11. $-\sqrt{2}+3\sqrt{5}$ **13.** $-\sqrt{2}+\sqrt{45}$

15. **a.** $\mathbb{Q}$ **b.** $\mathbb{Q}(\sqrt{6})$ **c.** $\mathbb{Q}$

17. $\mathbb{Q}(\sqrt{2}, \sqrt{3}, \sqrt{5})$ **19.** $\mathbb{Q}(\sqrt{3}, \sqrt{10})$ **21.** $\mathbb{Q}$

25. **a.** $3-\sqrt{2}$ **b.** They are the same maps.

27. $\sigma_3(0)=0, \sigma_3(1)=1, \sigma_3(2)=2, \sigma_3(\alpha)=-\alpha, \sigma_3(2\alpha)=-2\alpha,$
$\sigma_3(1+\alpha)=1-\alpha, \sigma_3(1+2\alpha)=1-2\alpha, \sigma_3(2+\alpha)=2-\alpha,$
$\sigma_3(2+2\alpha)=2-2\alpha; \mathbb{Z}_3(\alpha)_{\{\sigma_3\}}=\mathbb{Z}_3$

29. **a.** *F* **c.** *T* **e.** *F* **g.** *T* **i.** *T*

37. Yes