SECTION 49

1. The identity map of E onto E;
τ given by $\tau(\sqrt{2}) = \sqrt{2}$, $\tau(\sqrt{3}) = -\sqrt{3}$, $\tau(\sqrt{5}) = -\sqrt{5}$

3. τ_1 given by $\tau_1(\sqrt{2}) = \sqrt{2}$, $\tau_1(\sqrt{3}) = \sqrt{3}$, $\tau_1(\sqrt{5}) = -\sqrt{5}$;
τ_2 given by $\tau_2(\sqrt{2}) = \sqrt{2}$, $\tau_2(\sqrt{3}) = -\sqrt{3}$, $\tau_2(\sqrt{5}) = \sqrt{5}$;
τ_3 given by $\tau_3(\sqrt{2}) = -\sqrt{2}$, $\tau_3(\sqrt{3}) = \sqrt{3}$, $\tau_3(\sqrt{5}) = \sqrt{5}$;
τ_4 given by $\tau_4(\sqrt{2}) = -\sqrt{2}$, $\tau_4(\sqrt{3}) = -\sqrt{3}$, $\tau_4(\sqrt{5}) = -\sqrt{5}$

5. The identity map of $\mathbb{Q}(\sqrt[3]{2}, \sqrt{3})$ into itself;
τ_1 given by $\tau_1(\alpha_1) = \alpha_1$, $\tau_1(\sqrt{3}) = -\sqrt{3}$ where $\alpha_1 = \sqrt[3]{2}$;
τ_2 given by $\tau_2(\alpha_1) = \alpha_2$, $\tau_2(\sqrt{3}) = \sqrt{3}$ where $\alpha_2 = \sqrt[3]{2}(-1 + i\sqrt{3})/2$;
τ_3 given by $\tau_3(\alpha_1) = \alpha_2$, $\tau_3(\sqrt{3}) = -\sqrt{3}$;
τ_4 given by $\tau_4(\alpha_1) = \alpha_3$, $\tau_4(\sqrt{3}) = \sqrt{3}$ where $\alpha_3 = \sqrt[3]{2}(-1 - i\sqrt{3})/2$;
τ_5 given by $\tau_5(\alpha_1) = \alpha_3$, $\tau_5(\sqrt{3}) = -\sqrt{3}$;

7. **a.** $\mathbb{Q}(\pi^2)$ **b.** τ_1 given by $\tau_1(\sqrt{\pi}) = i\sqrt{\pi}$, τ_2 given by $\tau_2(\sqrt{\pi}) = -i\sqrt{\pi}$

SECTION 50

1. 2 **3.** 4 **5.** 2 **7.** 1 **9.** 2 **13.** $1 \le [E : F] \le n!$

15. Let $F = \mathbb{Q}$ and $E = \mathbb{Q}(\sqrt{2})$. Then

$$f(x) = x^4 - 5x^2 + 6 = (x^2 - 2)(x^2 - 3)$$

has a zero in E, but does not split in E.

23. **a.** 6

SECTION 51

1. $\alpha = \sqrt[6]{2} = 2/(\sqrt[3]{2}\sqrt{2}).\sqrt{2} = (\sqrt[6]{2})^3$, $\sqrt[3]{2} = (\sqrt[6]{2})^2$. (Other answers are possible.)

3. $\alpha = \sqrt{2} + \sqrt{3}$. $\sqrt{2} = (\frac{1}{2})\alpha^3 - (\frac{9}{2})\alpha$, $\sqrt{3} = (\frac{11}{2})\alpha - (\frac{1}{2})\alpha^3$. (Other answers are possible.)

7. $f(x) = x^4 - 4x^2 + 4 = (x^2 - 2)^2$. Here $f(x)$ is not an irreducible polynomial. Every irreducible factor of $f(x)$ has zeros of multiplicty 1 only.

15. **b.** The field F **c.** $F[x^p]$

SECTION 52

1. $\mathbb{Z}_3(y^3, z^9)$ **3.** $\mathbb{Z}_3(y^4, z^2)$

5. **a.** F **c.** F **e.** F **g.** T **i.** T

SECTION 53

1. 8 **3.** 8 **5.** 4 **7.** 2

9. The group has two elements, the identity automorphism ι of $\mathbb{Q}(i)$ and σ such that $\sigma(i) = -i$.

11. **a.** Let $\alpha_1 = \sqrt[3]{2}$, $\alpha_2 = \sqrt[3]{2}\frac{-1 + i\sqrt{3}}{2}$, and $\alpha_3 = \sqrt[3]{2}\frac{-1 - i\sqrt{3}}{2}$.

The maps are
ρ_0, where ρ_0 is the identity map;
ρ_1, where $\rho_1(\alpha_1) = \alpha_2$ and $\rho_1(i\sqrt{3}) = i\sqrt{3}$;