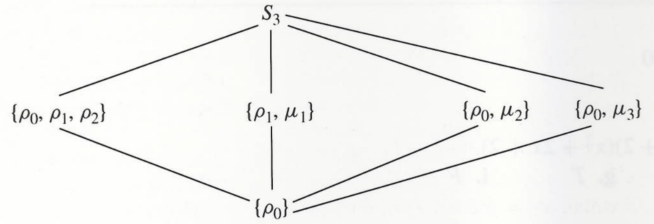
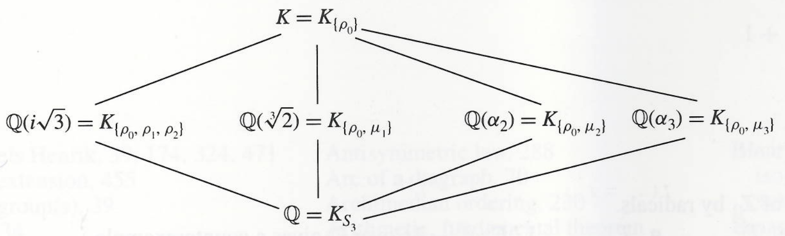
ρ_2, where $\rho_2(\alpha_1) = \alpha_3$ and $\rho_2(i\sqrt{3}) = i\sqrt{3}$;
μ_1, where $\mu_1(\alpha_1) = \alpha_1$ and $\mu_1(i\sqrt{3}) = -i\sqrt{3}$;
μ_2, where $\mu_2(\alpha_1) = \alpha_3$ and $\mu_2(i\sqrt{3}) = -i\sqrt{3}$;
μ_3, where $\mu_3(\alpha_1) = \alpha_2$ and $\mu_3(i\sqrt{3}) = -i\sqrt{3}$.

b. S_3. The notation in (a) was chosen to coincide with the notation for S_3 in Example 8.7.

c.



Group diagram



Field diagram

13. The splitting field of $(x^3 - 1) \in \mathbb{Q}[x]$ is $\mathbb{Q}(i\sqrt{3})$, and the group is cyclic of order 2 with elements: ι, where ι is the identity map of $\mathbb{Q}(i\sqrt{3})$, and σ, where $\sigma(i\sqrt{3}) = -i\sqrt{3}$.

15. **a.** *F* **c.** *T* **e.** *T* **g.** *F* **i.** *F*

25. *Partial answer:* $G(K/(E \vee L)) = G(K/E) \cap G(K/L)$

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3. $\mathbb{Q}(\sqrt[4]{2}, i)$: $\sqrt[4]{2} + i$, $x^8 + 4x^6 + 2x^4 + 28x^2 + 1$;
$\mathbb{Q}(\sqrt[4]{2})$: $\sqrt[4]{2}$, $x^4 - 2$;
$\mathbb{Q}(i\sqrt[4]{2}))$: $i(\sqrt[4]{2})$, $x^4 - 2$;
$\mathbb{Q}(\sqrt{2}, i)$: $\sqrt{2} + i$, $x^4 - 2x^2 + 9$;
$\mathbb{Q}(\sqrt[4]{2} + i(\sqrt[4]{2}))$: $\sqrt[4]{2} + i(\sqrt[4]{2})$, $x^4 + 8$;
$\mathbb{Q}(\sqrt[4]{2} - i(\sqrt[4]{2}))$: $\sqrt[4]{2} - i(\sqrt[4]{2})$, $x^4 + 8$;
$\mathbb{Q}(\sqrt{2})$: $\sqrt{2}$, $x^2 - 2$;
$\mathbb{Q}(i)$: i, $x^2 + 1$;
$\mathbb{Q}(i\sqrt{2})$: $i\sqrt{2}$, $x^2 + 2$;
$\mathbb{Q}$: 1, $x - 1$

5. The group is cyclic of order 5, and its elements are

	ι	σ_1	σ_2	σ_3	σ_4
$\sqrt[5]{2} \to$	$\sqrt[5]{2}$	$\zeta(\sqrt[5]{2})$	$\zeta^2(\sqrt[5]{2})$	$\zeta^3(\sqrt[5]{2})$	$\zeta^4(\sqrt[5]{2})$

where $\sqrt[5]{2}$ is the real 5th root of 2.