

§ 22 Rings of polynomials, cont.

How do we frame polynomial equations like

$$a_0 + a_1 x + \dots + a_k x^k = 0 \quad ?$$

R : ring (often field), $a_i \in R$
 $\nwarrow$ comm., ~~associ.~~ with mult. ident.

we say this eq. is defined over R

S : ~~comm.~~ ring that contains R

$\rightarrow$ system to look solutions for

Ex. $2 - x^2 = 0$ means $x^2 = 2$

this eq. is defined over $\mathbb{Z}$

has solutions in $\mathbb{C}$

but also: $\begin{bmatrix} 0 & 2 \\ 1 & 0 \end{bmatrix} \in M_2$ works as sol. if

we think $\begin{bmatrix} k & 0 \\ 0 & k \end{bmatrix}$ as a copy of $k \in \mathbb{Z}$

(We just need to start with ring hom. $R \rightarrow S$)

Thm (cf. 22.4) R, S as above; Then a choice of
 - ring hom $R[X] \xrightarrow{\phi} S$ compat. with inclusion $R \subset S$
 for the const. polynoms.

- an element $\beta \in S$

are the same thing up to the corresp. $\phi(X) = \beta$

Proof Idea: for any $\beta \in S$ we can def. a hom ϕ

by $\phi(a_0 + a_1 X + \dots + a_k X^k) = a_0 + a_1 \beta + \dots + a_k \beta^k$

and there is no ~~extra~~ extra choice for hom $R[X] \xrightarrow{\phi} S$

Concretely: writing $\phi = \phi_\beta$ for the above

- check that ϕ_β is a ring hom.

- compatibility with const. polynoms

(cont.) - given any hom $R[X] \xrightarrow{\psi} S$ compat. with const. polynom. emb.

$$\beta = \psi(X) \text{ satisfies } \psi = \phi_\beta$$

These can be verified by expanding definitions

Then : given a polynomial $f = f(X) \in R[X]$

- finding $\beta \in S$ s.t. $f(\beta) = 0$

- finding $\phi : R[X] \rightarrow S$ s.t. $\phi(f) = 0$

are the same problems

Following the idea behind $\mathbb{Z}_m = \{ \text{integers up to cong. mod. } m \}$

we can consider

" $R[X]_f$ " = { polynomials up to "cong. mod f " }
identify g, h when $f \cdot k = g - h$
for some k

then a ring hom " $R[X]_f \rightarrow S$ " would corresp.

to $\phi : R[X] \rightarrow S$ s.t. $\phi(f) = 0$

we will come back to this in § 26.

Digression algebras over a commutative ring

R : comm. ring. (often assume mult. identity)

an algebra over R (or R -algebra) is given by

- ring S (can be noncomm.)

- ring hom $\eta : R \rightarrow S$

s.t. $\eta(a)b = b\eta(a)$ for all $a \in R, b \in S$

Ex. $S = R[X]$ η : inclusion as const. polynoms.

$$S = M_n(R) = \{ (a_{ij})_{i,j=1}^n \mid a_{ij} \in R \}$$

Rem. Any ring with mult. id. can be considered
as a $\mathbb{Z}$ -algebra

Homomorphism of $\mathbb{Z}$ -algebras : $(S, \eta) \rightarrow (S', \eta')$

- ring hom $S \xrightarrow{\phi} S'$

- compat. with η & η' : $\phi(\eta(a)) = \eta'(a)$

So the above discussion can be phrased as

roots of polynom $f(X)$ in S

$\equiv$ $\mathbb{Z}$ -alg. homs. " $R[X]_f$ " $\rightarrow S$

1. The first part of the paper is devoted to the study of the

properties of the function

$f(x) = \sum_{n=0}^{\infty} \frac{x^n}{n!}$ and its derivatives.

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