

MOCK EXAM PROBLEMS FOR MAT2200

All problems have the same weight. Note: You must give full reasons for all your answers.

Problem 1. For each pair of groups among the following list, determine if they are isomorphic or not. If they are isomorphic, give an isomorphism. If they are not, give a reason why.

- $\{z \in \mathbb{C} \mid z^4 = 1\}$, with multiplication
- $\mathbb{Z}_4$
- $\mathbb{Z}_5$

Problem 2. Consider the following maps from $\mathbb{Z}^2$ to $\mathbb{Z}$. Determine if they are group homomorphisms or not.

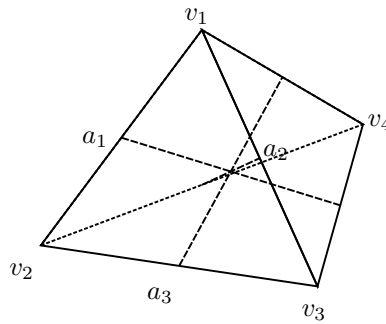
- addition map $f(x, y) = x + y$
- multiplication map $g(x, y) = xy$

Problem 3. Consider the subgroup $H \subset S_5$, given as

$$H = \{\sigma \in S_5 \mid \sigma(i) \in \{1, 2\} \text{ for } i \in \{1, 2\}, \sigma(i) \in \{3, 4, 5\} \text{ for } i \in \{3, 4, 5\}\}.$$

Determine if H is normal or not.

Problem 4. Consider the action of S_4 on tetrahedron, where each $\sigma \in S_4$ sends the vertex v_i to $v_{\sigma(i)}$ for $i = 1, 2, 3, 4$. Consider the subgroup of permutations that induce rotations around one of the axis a_1, a_2, a_3 . (We allow the trivial permutation too.) What is the order of this group?



Problem 5. Determine if the following rings are integral domains or not.

- $\mathbb{Z}_5$
- $\mathbb{Z}_6$

Problem 6. What are the maximal ideals of $\mathbb{Z}_6$?

Problem 7. Consider the polynomial $f(X) = X^2 + 1$ in $\mathbb{Q}[X]$. Determine if

$$\{f(X)g(X) \mid g(X) \in \mathbb{Q}[X]\} \subset \mathbb{Q}[X]$$

is a prime ideal or not.

Problem 8. Consider the polynomial $f(X) = X^2 + 1$ in $\mathbb{Z}_5[X]$. Determine if

$$\{f(X)g(X) \mid g(X) \in \mathbb{Z}_5[X]\} \subset \mathbb{Z}_5[X]$$

is a prime ideal or not.

Problem 9. What is the degree of extension of $\mathbb{Q}(\sqrt{5})$ over $\mathbb{Q}$?

Problem 10. Consider the extension $\mathbb{Q}(\sqrt[3]{3}) \supset \mathbb{Q}$. Show that this is not a splitting extension.

Problem 11. Let $f(X)$ be the minimal polynomial of $\sqrt[3]{3}$ over $\mathbb{Q}$. Determine the splitting extension $E \supset \mathbb{Q}$ for $f(X)$.

Problem 12. Determine the automorphism group of the extension $G(E/\mathbb{Q})$ for the extension $E \supset \mathbb{Q}$ above.