Summation factors and 2.1

In the book's notation you can freely choose s_0 and a_0, the resulting sequences will differ by scaling which cancels when solving for T_n.

In the book's solution for 2.1 $s_n = 2^{n-1}/n!$, the summation factor obtained by choosing $s_0 = 1 = a_0$. As an example let's instead chose $a_0 = 2$ (seems natural, so that $a_n = 2$ for all $n \geq 0$). Then $s_n = 2^n/n!$ and the expression for T_n then becomes:

$$\begin{aligned} T_n &= \frac{n!}{2^{n+1}}\left(\sum_{k=1}^{n} \frac{2^k}{k!} 3k! + 6\right) \\ &= \frac{3n!}{2^{n+1}}\left(\sum_{k=1}^{n} 2^k + 2\right) \\ &= \frac{3n!}{2^n}\left(\sum_{k=1}^{n} 2^{k-1} + 1\right) = 3n! \end{aligned}$$

(recall that $1 + 2 + 2^2 + \cdots + 2^{n-1} = 2^n - 1$).

1.17

The probability that a set contain adjacent integers is the number of sets with at least one pair of adjacents divided by the total number of sets. We can represent any choice of 6 numbers from a given set $\{1, 2, \ldots, n\}$ by an *ordered* 6-tuple $(a_1, \ldots, a_6)$, $a_i < a_{i+1}$ (there are $\binom{49}{6}$ of these). The ordered tuples *without* any adjacent integers are in bijection with arbitrary ordered tuples from $\{1, 2, \ldots, 49 - 5\}$ with the map:

$$(a_1, \ldots, a_6) \mapsto (a_1, a_2 - 1, \ldots, a_6 - 5)$$

(the condition that none of the a_i are adjacent can be expressed as $a_i < a_{i+1} - 1$, so the map sends an ordered increasing tuple to an ordered increasing tuple). Therefore there are $\binom{44}{6}$ sets in $\{1, 2, \ldots, 49\}$ *without* adjacent integers (so $\binom{49}{6} - \binom{44}{6}$ *with*) and the probability of choosing a set *with* two adjacent is $1 - \binom{44}{6}/\binom{49}{6}$.

2.5

Induct on the number of columns n. If we can rearrange the rows so that the *first* column contains one of each colored stone, then we've reduced the problem to the $n - 1$ case (having reduced the columns to consider by one, as well as the number of each colored stone by 1).

Here's one way to show that we can always do this: suppose (WLOG) that the stone in the first row/first column is red, then in the second column there's at least one stone that is *not* red (only n of each stone), say it's white. Rearrange the second row in any way so that this white stone is in the first column. If the third column contains a green stone then we can move it to the first column and we are done. Otherwise it must contain only red and white stones, and therefore each of the first two columns contain at least one green. Say the first stone in the 3rd row is red, then we can arrange the first row so that it starts with it's green stone.