1.57

There is some ambiguity in the book about the "event A". We want to consider the event that a set of edges among exactly k-people (i.e. a "pattern of acquaintance") form a complete graph. Then we see $p(A) = 2^{-\binom{k}{2}}$ since there are $\binom{k}{2}$ possible edges between the k people, each having probability $1/2$ of being in a random pattern of acquaintance.

See also Proofs from the book, p.263.

2.3

The sum notation could be rewritten:

$$f(j,n) = \sum_{i \leq j \leq k \leq n} k$$

Recall that the *triangular numbers* $T_n = \sum_{k=1}^{n} k = \frac{n(n+1)}{2} = f(0,n)$. We get $f(j,n) = T_n - T_j$.

2.12

Choosing a divisor of an integer $n = p_1^{a_1} p_2^{a_2} \cdots p_k^{a_k}$ (its prime factorization) is equivalent to choosing a divisor for each of its prime power factors, equivalently an integers $0 \leq x \leq a_i$ for each p_i. Therefore n has $(a_1 + 1) \cdots (a_k + 1)$ divisors.

A number with 28 divisors has prime factorization of the form $p_1 p_2 p_3^6$ or $p_1^3 p_2^6$ or $p_1 p_2^{13}$ or p_1^{27}. The smallest integer in one of these forms is $2^6 \cdot 3 \cdot 5 = 960$.