2.10

Use inclusion-exclusion. First, there are $6! = 720$ total permutations of 'EHIRSW'. One way to count the sequences (permutations) containing 'WIR' for example is the following:

Any way we insert the remaining letters E H S into the sequence WIR forms a sequence containing WIR (and all sequences containing WIR can be realized this way). Counting the ways to add them in a specified order avoids over counting. For example, there are 4 places to insert E into WIR, 5 places to insert H into the resulting sequence which then leaves 6 places to insert S. Thus the number of sequences containing WIR (or IHR or SIE) is $4 \cdot 5 \cdot 6 = 120$

We can use the same idea to count the sequences containing two given subsequences (say IHR and SIE). There are 3 minimal sequences containing IHR and SIE: 'SIHRE', 'SIEHR' and 'SIHER'. All sequences containing both IHR and SIE can be obtained by inserting the remaining letter W into exactly one of these minimal sequences, so there are $3 \cdot 6$ such sequences. The minimal sequences containing WIR and SIE are 'WSIER' 'WSIRE' 'SWIER' and 'SWIRE' (admitting $4 \cdot 6$ sequences), and the only minimal sequence containing WIR and IHR is 'WIHR' (admitting $5 \cdot 6$ supersequences)

Finally, there are 6 sequences containing all three 'WIR' 'IHR' and 'SIE': 'SWIEHR', 'SWIHER', 'SWIHRE', 'WSIEHR', 'WSIHER', 'WSIHRE'.

Thus the number of sequences containing none is:

$$720 - 3 \cdot 120 + 12 \cdot 6 - 6 = 426.$$

Can check:

```
import itertools
string = "ehirsw"
permutations = list(itertools.permutations(string))
hasNone = 0
for c in permutations :
    leaveWIR = tuple(char for char in c if char in ('w', 'i', 'r'))
    leaveIHR = tuple(char for char in c if char in ('i', 'h', 'r'))
    leaveSIE = tuple(char for char in c if char in ('s', 'i', 'e'))
    if  leaveWIR != ('w', 'i', 'r') and leaveIHR != ('i', 'h', 'r') and leaveSIE !=
('s', 'i', 'e'):
        hasNone += 1
print(hasNone)
```

2.11

Note that counting n powers less than or equal to m, i.e. integers $1 \leq a \leq m$ such that there is $b \in \mathbb{Z}$ with $b^n = a$, is the same as counting b such that $b \in \mathbb{N}$ and $b^n \leq m$. For example, one million $= 10^6 = (10^3)^2$ and we see that there are 10^3 squares less than or equal to one million.

Recall that a is a square and a cube if and only if a is a 6th-power.