MAT2250 Exercise list
Spring 2024 - Instructor: Kris Shaw

11. Week of April 22nd - Coding theory

Warm ups.

(1) What is the Hamming distance between the code words 0010 and 0100?
(2) What is the maximum Hamming distance between two code words in $\{0,1\}^n$?
(3) For which t is the code $\{0000, 1100, 0011\}$ t-error correcting and t-error detecting?
(4) What is the maximum size of a code $C \subseteq \{0,1,2\}^4$ with $d(C) \geq 3$?
(5) Can a code $C \subseteq \{0,1\}^n$ with $|C| = 9$ be linear, where $\{0,1\} = GF(2)$.

Problem 11.1. Show that for a 2-error correcting code $C \subseteq \{0,1\}^8$ we have $|C| \leq 4$. Does there exist a code with $|C| = 4$? Hint: Look for a linear one!

Problem 11.2. Let $(S, \mathcal{B})$ be a t-design with $|S| = v$, $|B| = k$, for all $B \in \mathcal{B}$ and $\lambda = 1$. Define a code

$$C = \{c_B \mid B \in \mathcal{B}\} \subseteq \{0,1\}^v,$$

where c_B is the vector of zeros and ones with i-th coordinate equal to 1 if $i \in B$ and 0 otherwise.
What is $|C|$? Show that $d(C) \geq 2(k - t + 1)$. Hint: $d(c_{B_1}, c_{B_2})$ is the size of the symmetric difference of the sets B_1 and B_2.

Problem 11.3. Prove that the Hamming distance satisfies the triangle inequality, namely

$$\Delta(a, c) \leq \Delta(a, b) + \Delta(b, c)$$

for all $a, b, c \in A^n$.

Problem 11.4. Consider the vectors $c = (0, 1, 1, 2)$ and $c' = (1, 0, 1, 1)$ in $GF(3)$. The linear code $C \subseteq GF(3)^4$ generated by c and c' consists of all linear combinations over $GF(3)$ of c and c'. Write down the list of code words in C and verify that $d(C) = 3$. Conclude that C is 1-perfect.