# MAT2250 Exercise list
# Spring 2024 - Instructor: Kris Shaw

## 11. Week of May 6th - Linear and cyclic codes

**Warm ups.**

(1) Write down the polynomial for the code word $(1, 2, 0, 1, 2) \in GF(3)$.
(2) Is the binary code $\{0000, 1100, 0110, 0011\}$ cyclic?
(3) What is the dimension of the cyclic code $C$ with generating polynomial $g(x) = x - 1$?
(4) Is there a cyclic code $C \subset GF(3)^5$ with generating polynomial $g(x) = x + 1$?

**Problem 11.1.** Consider the linear code $C \subseteq GF(3)^5$ given by the check matrix

$$H = \begin{pmatrix} 1 & 0 & 2 & 0 & 0 \\ 0 & 1 & 0 & 2 & 0 \\ 0 & 0 & 1 & 1 & 1 \end{pmatrix}$$

What is the generating matrix of $C$ ?
Show that $C$ is 1-error correcting. Suppose the message 21012 was received over a channel with at most one error. Find the syndrome and correct the error.

**Problem 11.2.** Show that in a binary linear code, either all codewords have even weight or half have even weight and half odd weight.

**Problem 11.3.** Let $C \subseteq GF(2)^7$ be the cyclic code generated by $g(x) = x^3 + x^2 + 1$. Find the generating matrix and the partity check matrix of $C$. Is the word $c = (1, 1, 1, 0, 1, 0, 1)$ in $C$? Find the syndrome of $c$.

**Problem 11.4.** Show that the dual code of a cyclic code is cyclic.

**Problem 11.5.** Let $C_1$ and $C_2$ be two cyclic codes over $GF(q)$ with generator polynomials $g_1(x)$ and $g_2(x)$, respectively. Show that $C_1 \subseteq C_2$ if and only if $g_2(x)|g_1(x)$.