MAT2250 Exercise list
Spring 2024 - Instructor: Kris Shaw

8. Week of March 11th - Matchings, Circuits, and Traveling Salesperson Problem

Warm ups.

(1) Can you remove 3 edges from $K_{4,4}$ so that is no longer a matching with $|M| = 4$?
(2) For what values of n is K_n Eulerian?
(3) For what values of n and m is $K_{n,m}$ Eulerian?
(4) How many Hamiltonian cycles are there in K_n?

Problem 8.1. Suppose a bipartite graph is k regular for $k \geq 1$. Show that $|S| = |T|$ and that G contains a matching with $|M| = |S| = |T|$.

Problem 8.2. Use the Hungarian algorithm to find a perfect matching of $K_{5,5}$ of minimal weight if the weight function on the edges is encoded by the matrix:

$$\begin{pmatrix} 8 & 7 & 5 & 11 & 4 \\ 9 & 7 & 6 & 11 & 3 \\ 12 & 9 & 4 & 8 & 2 \\ 1 & 2 & 3 & 5 & 6 \\ 11 & 4 & 2 & 8 & 2 \end{pmatrix}.$$

Problem 8.3. Describe a Hamiltonian cycle in the hypercube Q_n for all n.

Problem 8.4. Let (c_{ij}) be a symmetric cost matrix for a TSP with $c_{ij} > 0$ for $1 \leq i, j \leq n$. Suppose the TSP satisfies the triangle inequality: $c_{ij} + c_{jk} \geq c_{ik}$ for all i, j, k. Consider the following algorithm for finding a Hamiltonian circuit. Start at an arbitrary vertex v and write $C_1 = \{v\}$. Suppose at the k-th step the circuit $C_k = v_1, \ldots, v_k$ has been constructed. Find the vertex u with minimal distance to any vertex in C_k, and insert u into the circuit before the corresponding vertex with minimal distance to u. Show that for the tour T just constructed that we have $c(T) \leq 2c(T_{opt})$.